Does it pay to voluntarily disclose private information?

Dima Leshchinskii*

Department of Finance and Economics, Groupe HEC

1 rue de la Liberation

78351 Jouy-en-Josas Cedex, FRANCE

First draft: October 2000

Current version: June 2001

*e-mail: leshchinskii@hec.fr. Phone: +33 1-39-67-94-10, fax: +33 1-39-67-70-85. The author would like to thank Neil Brisley, Bernard Dumas, Thierry Foucault, Paolo Fulghieri, Dmitry Lukin, Matti Suominen and seminar participants at Boston College, Groupe HEC, University of Notre Dame, University of Wisconsin, Madison, and at the EFMA conference in Athens for their valuable comments.

Does it pay to voluntarily disclose private information?

Abstract

This paper studies how strategic interaction between players can influence their decisions as to whether to acquire information and whether to reveal their private information to others. We show how a player can increase his utility by disclosing part of his private information, when such disclosure stimulates others to produce new information that is useful for him. We derive conditions for information disclosure to be the equilibrium strategy and solve for the equilibrium.

When both traders are risk-neutral they each increase their profits by specializing in the types of information they acquire. This specialization encourages the traders to share information directly or through trades. When the traders have different risk preferences, the more risk averse trader may prefer to reveal his less precise information in order to stimulate information production by less risk averse traders. This strategy can give him higher utility than if he merely acquired information by himself.

JEL codes: D82; D83; G12; G14

A client receives a call from his broker, who says that he has some private information about company X, but he will not disclose it. The client makes an effort to collect more information, learns something really interesting about the company and then buys or sells the company's shares, perhaps even through another broker. Was the information that he produced the same as the broker had? Why did the broker give a hint? Is this story consistent with existing market microstructure models where the traders usually act as adversaries competing with each other (e.g., Foster and Viswanathan (1996))? In such models, traders build their strategies to avoid disclosure of private information to other traders. This competition can impede information revelation through trading if the information is exogenously given and can even halt the information production process altogether (Grossman and Stiglitz (1980)).

In this paper we show that traders may be better off by acting more cooperatively and disclosing part of their private information. While models of voluntary information disclosure do exist in the literature(recent examples include Admati and Pfleiderer (2000) and Boot and Thakor (2001)), the novelty of our paper is that this disclosure triggers new information production, which both traders benefit from. If the original information has not been revealed, new information would have never been produced and traders would not make additional profits.

The intuition behind the voluntary disclosure is the following: a privately informed agent discloses the part of his private information which is sufficient to induce the information production by other agents if, as a result, the agent obtains an expected utility greater than is available from his alternative actions, e.g., from nondisclosure or from additional information production by himself. In other words, by announcing some news the agent triggers an information production cascade, which is to his own benefit. As an analogy, consider a jigsaw puzzle: In this game, different players cannot connect their own pieces together before some missing pieces are placed by others. By placing some of his pieces, a player gives others a chance to place more of theirs, thus allowing himself to finish the picture.

In a simplified way the process of mutually beneficial information sharing can be described as follows. There are two agents, A and N, who have access to different information production technologies. Agent A is endowed with a relatively primitive technology, while agent N has access to a more advanced information production technology. However, this technology is expensive and without additional information agent N will not use it. If trader A reveals part of his information, then agent N can revise his decision about producing the information (acquiring a signal). Whenever revealed information makes agent N's information production technology profitable, he will use it. Agent A is able to infer the new results from some publicly observed signal (such as market prices) and use it to his own benefit.

The main contributions of the paper are the following. First, using a market microstructure model with several traders, we find conditions that determine when information disclosure by one agent is the equilibrium strategy leading to new information discovery, which both agents benefit from. We prove that for a wide range of parameters of the model this is the only pure strategy equilibrium. In other cases, either this equilibrium is Pareto dominating or no other information production equilibrium is possible. Second, we find an analytical solution for the equilibrium and show how the traders endogenously choose the information precision of the signals they buy.

The model has a risky security whose value is subject to an information shock. There are three types of players — one competitive market maker and two types of traders. We consider two cases: 1) when all agents are risk-neutral, and 2) when the agents have heterogeneous preferences. In the first case, the incentive to reveal private information is driven by unequal information production capabilities of the different types of traders. A signal of the same precision costs more to one trader than to another. In the second case, the traders have different risk preferences. The first type is a risk averse trader, endowed with a primitive information production technology, which allows him to observe whether an innovation has occurred or not. The second type is a risk neutral trader (indeed there may be more than one risk neutral trader). There exists a more expensive information production technology which allows its user to observe more precise signals about the security's true value, given that the innovation has occurred. However, without knowledge of the occurrence of the information shock, the risk neutral type is not interested in using the technology, because it is too expensive. Only when he is sure that an innovation has occurred, is it worth his while to invest in producing a more precise signal. One of the possible equilibrium strategies for the risk averse trader is to reveal his private information about the innovation in order to stimulate information production by the risk neutral trader(s). This strategy is the only pure strategy equilibrium if the information production costs are in the intermediate range. If the cost is below this range, then information sharing can still dominate the ”no announcement”strategy, when the risk averse trader acquires the signal himself. When the cost is above the range, then the risk averse trader can commit not to acquire information and the market maker will keep the price of the risky security constant allowing the risk averse trader to get rid of all his risky endowment.

When there is more than one risk neutral trader, the risk averse trader becomes better informed than anybody else. This happens because his partition includes information about the system's noise. We know from existing models of informed trading (e.g., Grossman and Stiglitz (1980), Kyle (1985) and Glosten (1989)) that noise is necessary to prevent full information revelation. In our model, as in Glosten (1989), the endowment shock experienced by a risk-averse trader serves as a source of such noise — the insider trades both for information and risk sharing reasons. Knowing his own order, the risk averse trader is able to filter out noise from the price and gets a very precise signal about the security's value. A fairly similar situation happens in Brunnermeier (1998), where the least informed agent is able to manipulate the price. In our model, the initially least informed agent suddenly becomes the best informed! Therefore, when making the information acquisition and information disclosure decisions, the agents take into account how it will influence the actions of other players and the final outcome. When producing information, the agents consider the following: 1) how the information suits their needs, 2) what the information costs and 3) what interaction with other players it will lead to. Differences in information production technologies, in information costs for different players, and in individual preferences, e.g., heterogeneous risk attitudes, create incentives for mutually beneficial information sharing and “joint” information production.

Traditional Kyle-style models fail to take this into account, since they take the information structure as exogenously given. While these models explicitly show how the traders can gain from information-driven trading, they do not explain how the traders choose what kind of information they want to acquire.

Since information is costly, traders choose to buy signals which best fit their needs (maximize their utility functions). Verrecchia (1982) studies the information acquisition decisions made by atomistic risk-averse traders in a rational expectations equilibrium model. The traders in his model behave competitively and ignore the effect of their own decisions on the actions of other traders. Verrecchia shows that the level of signal precision that a trader decides to buy is determined by his risk tolerance and other parameters of the system. Signal precision is a non-decreasing function of risk tolerance, a non-increasing function of price informativeness and, hence, a non-decreasing function of noise in the system (variance of supply of risky stock). Persico (1995) demonstrates a similar result that if signal precision is the variable of choice, then a more risk averse trader might buy a lower precision signal. More specifically, more risk-averse traders are interested in buying cheaper and less precise information, which decreases their uncertainty about the overall future payoff on the entire portfolio, whereas less risk-averse traders are more interested in precise information, which gives them better estimates of individual security payoffs, even if these signals are more expensive. Both types are interested in driving down the price they pay for signals of specific precision. Obviously, one way of keeping costs low would be to avoid unnecessary costs associated with redundant signals and to select only those signals that contain information which cannot be extracted from observed prices.

Another factor determining the information acquisition decision is that decision's potential effect on the strategies of other traders. In this paper we build a model which describes how market participants interact in information production.

The trading part of our model synthesizes Kyle (1985), where the trader is risk-neutral and Glosten (1989) and Biais and Rochet (1996), where the trader is risk-averse. Medrano and Vives (1997) also have two types of traders, but in their model the risk-averse traders are atomistic and act competitively.

In our model, both players act strategically when deciding whether or not to share information and what signal they want to acquire, whereas in the quoted papers information is costless, its precision is exogenously given and no effects of interaction between different traders behavior are taken into account.

Models with voluntary information disclosure are not completely unknown in the existing literature. For example, Gal-Or (1985), Gal-Or (1986) and Vives (1985) study the incentives to share private information either about uncertain common market demand or about unknown individual production costs. With Cournot competition traders reveal information about their private production costs and with Bertrand competition they are better off by pooling information about the market demand. Admati and Pfleiderer (2000) study voluntary disclosure by firms in financial markets. They focus on externalities that arise when firm values are correlated and the disclosures made by one firm affect the valuation of other firms. In Admati and Pfleiderer (1988) an information owner decides between selling information to other traders and trading strategically on it alone. His choice depends on the risk tolerance of the traders and precision of the information. If the information owner is risk-neutral, then he does not sell and simply trades on his own account, because by selling information he would create competition and reduce his own profits. For a risk averse seller it is profitable to sell the information, because it allows better risk sharing.

Boot and Thakor (2001) study what kind of information and how much of it firms should voluntarily disclose. Our model is somewhat related to the types of disclosure in their paper associated with complementary information.

Crawford and Sobel (1982) describe a model in which a better informed sender sends a (possibly) noisy signal to a receiver, who then takes an action that determines the welfare of both. Equilibrium signaling takes a simple form in which the sender partitions the support of the variable representing his private information and reports only which element of the partition his observation lies in. The sender's optimal strategy reflects a compromise between including enough information in the signal to induce the receiver to respond to it and holding back enough so that his response is as favorable as possible.

Other related literature includes Admati and Pfleiderer (1987) and Persico (1995), who study signal aggregation and complementarity of information. Persico (1995) also investigates the value of information in decision making problems. Foster and Viswanathan (1996) have a model of strategic interaction between players, but in their model the traders have similar preferences and compete with each other, whereas we discuss potential strategic cooperation in information processing.

The remainder of the paper is organized as follows. Section 1 describes the model with risk-neutral agents. Section 2 presents a more general model with traders who have heterogeneous risk preferences. Section 3 considers a particular case with just one risk neutral trader and Section 4 concludes.

1. The Model with Risk-Neutral Agents

Our model has the following components:

Securities

We have one riskless and one risky security. The payoff of the riskless security is normalized to one. With probability, q, the risky security suffers an innovation (information shock) at the beginning. If there is no innovation, then the security's payoff is $V_0 = \mu$ at the end. If there is an innovation, then its payoff, V, is normally distributed $V \sim N(\mu; \sigma^2)$, $\sigma^2 = \frac{1}{r}$. The probability, q, and the parameters of the distribution are public knowledge at the beginning of the period and the realization of V becomes public knowledge at the end.

Technically speaking, the payoff of the risky security is not normally distributed. However, this does not constitute a problem for the model's tractability, because we will show that trades do not occur if no innovation has taken place.

Agents

There are four agents — a competitive risk-neutral market maker, risk-neutral traders A and N who have access to information production technologies and noise traders. The traders can borrow and lend at the riskless rate, which is normalized to zero. During the trading period the traders A and N can submit demand schedules (limit orders). Noise traders' demand w is random and does not depend on price. We assume that $w \sim N(0; \sigma_w^2)$. We assume that trader A can observe w before submitting his limit order.

The market maker observes the combined value of a all orders. He also knows the number of traders, who submitted the orders, but cannot observe the individual orders. This setup is standard for the Kyle-style models. The market maker sets the price equal to the conditional expectation of the security's value.

Traders A and N maximize their utility functions (i.e., profits) by making information acquisition decisions and by choosing the orders they submit. They act strategically, taking into account the effect of their decision on the actions of other traders and the market maker's pricing decision.

Information Structure and the Timeline

If an innovation takes place, trader A observes a perfect binary signal

$$\varphi = \begin{cases} 0 & \text{if no innovation occurred,} \\ 1 & \text{if innovation occurred.} \end{cases}$$

Hence, trader A has perfect knowledge of whether the innovation has taken place or not, but he does not know the value of the innovation. He merely knows security's prior distribution, $N(\mu, \sigma^2)$, given that the innovation has taken place. This means that trader A knows only that the security's value has changed, but initially he has no other information about this change.

We call the period when information shock occurs t_0. At t_0 trader A can make an announcement (truthful or not) whether the innovation has occurred or not to some traders.

If a trader decides to trade, he informs the market maker about his intention at the beginning of period t_1. By the end of t_1 the market maker and those traders, who have decided to trade, know the total number of participating traders, m. There is no entry fee or any other transaction fee that participating traders have to pay.

During period t_1 trader A and traders N_i have access to information production technologies that can produce signals, S_j, $j = \{A, N_i\}$. These signals are costly with their cost $c_j(r_j)$ being dependent on the trader's type and on the signal precision, r_j.

Assumption 1. *We assume that* $c_A(r) \gg c_N(r)$

We also assume that $c_j(r)$ are increasing functions and that their exact form are common knowledge. At this point, we do not make any additional assumptions concerning the specific shape of the cost functions. They may be continuous or they may be a discrete-valued functions. In the latter case the traders have simply to choose a pair $(r_j; c(r_j))$ from a given menu.

If no innovation has occurred, then $S_j = \mu$ for any trader who invests in a precise signal. In this case the precise signal is redundant for trader A, because he already knows that $V = \mu$. If an innovation did occur, then $S_j = V + \varepsilon_j$, where ε_j is Gaussian noise, uncorrelated with the value of the innovation, V. That is, $\varepsilon_j \sim N\left(0;\ \sigma_j^2\right);\ \sigma_j^2 = \frac{1}{r_j}$. $j = \{A;\ N\}$.

Assumption 2. *For analytical tractability we assume that if $r_A \leq r_N$ then the signal observed by trader A is a noisier version of a signal observed by trader N, $\varepsilon_A = \varepsilon_N + \zeta$, where ζ is a normally distributed random variable independent of ε_N.*

This assumption does not change the results, it simply makes the computations analytically more tractable — whenever trader A's signal is less precise than the signal observed by trader N, trader A can ignore his own signal if he can observe or infer the value of signal S_N.

If a trader acquires the signal, he must pay for it regardless whether an innovation has taken place or not.

At time t_2, the traders, who participate in the trade, submit their demand schedules as functions of price. Trader A submits demand schedule $x(p)$ and trader N submits demand schedule $z(p)$. As we have mentioned, the number of participating traders, m, is known to the market maker and to the participating traders themselves if they decided to engage in information production. As in Kyle, the market maker cannot observe the individual orders and knows only the combined demand schedule, $y(p)$, $y(p) = x(p) + z(p) + w$. Although the traders do not observe y, they can infer it if the price function is monotone. The market maker sets the price equal to the conditional expectation given the information he has, $\Phi_{MM} = \{m; y\}$.

At time t_3, the true value of the risky security is revealed, the traders get their payoffs and the game ends.

The decision tree for players A and N is shown on Fig 1. For the case in which the innovation takes place, we can summarize traders' actions in the following table:

t_0	t_1 (signal acquisition)	t_2 (trading)	t_3
A observes φ A decides on announcement	1) A and N_i make trading decisions 2) the number of potential traders becomes known 3) traders acquire signals	traders submit demand schedules	payoff V

Trader A wants to maximize his wealth. Essentially, trader A faces a choice between the following options: 1) he can acquire a signal, $S_A = V + \varepsilon_A$, $\varepsilon_A \sim N\left(0; \sigma_A^2\right); \sigma_A^2 = 1/r_A$ and trade on this information; 2) he can make an announcement about the innovation, thus stimulating information production by trader N and then submit the demand schedule (limit order) incorporating trader's N information; 3) without acquiring information and announcing he can trade against noise traders' demand w; and 4) he can do nothing, refraining from trading at all.

If we assume that trader A chooses to refrain from trading whenever he is indifferent between trading and not trading, then the third option is never the equilibrium outcome. If he does not make an announcement about the innovation and acquires himself, then he incurs the information acquisition costs, but increases his profits from trading. When the information cost is low, trader A acquires the signal himself. But as the cost gets bigger, trader A becomes less willing to acquire information himself and more inclined to stimulate the information production by trader N.

1.1. Innovation Announcement Equilibrium

We are interested in finding conditions for the Nash equilibrium, when trader A truthfully announces about the innovation and trader N acquires signal S_N and participates in the trade. The game tree is depicted on Figure 1. The dotted lines connect the nodes which belong to the same information set for a given trader. If trader A conceals his information about the innovation, then at time t_0 trader N does not know whether the innovation has occurred or not. If innovation has taken place and trader A conceals this information, then at time t_1 he does not know whether trader N has acquired a signal or not.

Before the trading takes place, trader A makes two moves. In his first move he chooses whether to make an announcement or to conceal his information about the innovation. His announcement is not required to be truthful — when announcing, he can make a false statement, claiming that the innovation has taken place, when it has not and vice versa. In the second move trader A chooses whether or not to acquire a signal. Trader N makes only one move, when he makes an information acquisition decision.

Formally we denote trader A's strategy as $T_A = (T_{A,1}; T_{A,2})$ where the possible moves are $T_{A,1} = \{\textit{announce}, \textit{conceal}, \textit{falsify}\}$ and $T_{A,2} = \{S_A; \overline{S_A}\}$. For trader N, the strategy is $T_N = \{S_N; \overline{S_N}\}$. Here S_j means that trader j decides to acquire a signal and $\overline{S_j}$ means that trader j does not acquire a signal.

We will find conditions, under which in the linear equilibrium trader A truthfully makes an announcement about the innovation and remains uninformed, while trader N acquires a signal, S_N, incurring the signal acquisition cost. We assume that traders have access to different information production technologies, therefore it would be more expensive for trader A to acquire the same signal. When both traders A and N submit demand schedules (limit orders), then at the time of trade execution, t_3, both traders have the same information set, because trader N correctly infers the noise traders' demand and trader A correctly infers the value of S_N. Therefore, trader A faces a trade-off between saving money on the information acquisition costs and getting reduced profits from trade by sharing them with trader A.

Depending on the information cost function and other parameter values, other can equilibria exist, in which trader A conceals or falsifies information.

In order to derive the equilibrium conditions, we have to find traders' costs and benefits from different actions. If trader A is successful in averting trader N's from acquiring information and trading, then trader A has to acquire information himself. He pays the cost, $c_A(r_A^*)$, to acquire buy the signal, S_A^*, choosing precision, r_A^*, that would maximize his profits.

Proposition 1.1. *If trader A acquires signal S_A^* and trades alone by submitting the limit order x, then the pricing schedule is equal to*

$$p(y) = \mu + \lambda_A(x + w),$$

where

$$\lambda_A = \frac{1}{\sigma_w}\sqrt{\frac{r_A^*}{rR_A^*}}$$
$$R_A^* = r + r_A^*$$

and trader A's demand schedule x is

$$x = \frac{(S_A^* - \mu)}{2}\sigma_w\sqrt{\frac{rr_A^*}{R_A^*}} - \frac{w}{2} \tag{1.1}$$

Proof. *The proof is in Appendix II.*

Corollary 1.2. *If trader A trades alone, his expected profit is*

$$\Pi_A = \frac{1}{2}\sigma_w\sqrt{\frac{r_A^*}{rR_A^*}} - c_A\left(r_A^*\right)$$

Proof. *The proof is straightforward. We calculate trader A's expected profit using expression (1.1) for his limit order and expression (7.3) for coefficient λ of the pricing function, p.*

Choice of r_A^*

Trader A's choice of the signal precision r_A^* depends on the cost of the signal, $c_A(r_A)$. If $c_A(r_A)$ is convex and increasing, then r_A^* is determined as a solution to equation

$$\frac{\sigma_w\sqrt{r}}{4\sqrt{r_A}\left(r + r_A\right)^{3/2}} - c_A'(r_A) = 0.$$

subject to

$$\frac{1}{2}\sigma_w\sqrt{\frac{r_A}{rR_A}} - c_A\left(r_A\right) \geq 0.$$

Since $c_N(r_N)$ is convex $(c''(r_N) > 0)$ the second order condition is always satisfied.

Trader A trades alone whenever trader N refrains from trading. This happens in the case, when, given the information, available to him, trader N expects to suffer a net loss, if he decides to acquire signal, S_N, and trade. If trading becomes profitable for trader N, then both traders submit limit orders. If trader A knows for sure that trader N is acquiring the signal and trading, then he is better off by not buying signal S_A, because he can infer the value of S_N from the equilibrium price and , therefore, save the acquisition cost $c_A(r_A)$.

Proposition 1.3. *If trader N acquires signal S_N^* and both traders submit the limit orders, then the pricing schedule is equal to*

$$p(y) = \mu + \lambda_N y, \tag{1.2}$$

where

$$\begin{aligned} \lambda_N &= \frac{1}{\sigma_w}\sqrt{\frac{2r_N^*}{rR_N^*}} \\ y &= x + z + w, \end{aligned} \tag{1.3}$$

trader A's demand schedule, x, is

$$x = \frac{p-\mu}{4}\sigma_w\sqrt{\frac{2rR_N^*}{r_N^*}} - \frac{w}{2}$$

and trader N's demand schedule, z, is

$$z = \frac{(S_N-\mu)}{2}\sigma_w\sqrt{\frac{2rr_N^*}{R_N^*}} - \frac{p-\mu}{2}\sigma_w\sqrt{\frac{2rR_N^*}{r_N^*}}$$

Proof. *The proof is in Appendix II.*

Corollary 1.4. *Demands of traders A and N are identical*

$$x = z = (S_N - \mu)\,\frac{1}{3}\sigma_w\sqrt{\frac{2rr_N^*}{R_N^*}} - \frac{w}{3} \tag{1.4}$$

Corollary 1.5. *The ex-ante expected profits of traders A and N when they both trade are*

$$\Pi_A = \frac{1}{6}\sigma_w\sqrt{\frac{2r_N^*}{rR_N^*}}$$

for trader A and

$$\Pi_N = \frac{1}{6}\sigma_w\sqrt{\frac{2r_N^*}{rR_N^*}} - c_N\left(r_N^*\right)$$

for trader N, respectively

Proof. *To get the expected profits we have to substitute expression (1.4) for the limit orders and expressions (1.2) and (1.3) for the pricing function into maximization programs (7.4) and (7.5) and find the unconditional expected values.*

Choice of r_N^*

Trader N's choice of the signal precision r_N depends on the cost of the signal, $c_N(r_N)$. If $c_N(r_N)$ is convex and increasing, then r_N^* is determined as a solution to equation

$$\frac{\sigma_w\sqrt{2r}}{12\sqrt{r_N}\left(r + r_N\right)^{3/2}} - c'(r_N) = 0.$$

subject to

$$\frac{\sigma_w\sqrt{r_N}}{3\sqrt{2rR_N}} - c_N(r_N) \geq 0.$$

Since $c_N(r_N)$ is convex $(c''(r_N) > 0)$ the second order condition is always satisfied.

Properties of r_N, Π_N and Π_A

From

$$c'(r_N)\sqrt{r_N}\,(r+r_N)^{3/2}=\frac{\sigma_w\sqrt{2r}}{12} \tag{1.5}$$

by differentiation of an implicit function we find that for optimal r_N

$$\frac{\partial r_N}{\partial \sigma_w}=\frac{\sqrt{2rr_N}}{6\sqrt{(r+r_N)}\,[2r_N\,(r+r_N)\,c''(r_N)+(r+4r_N)]}>0,$$

$$\begin{aligned}\frac{\partial r_N}{\partial r} &= \frac{\left[\frac{\sigma_w\sqrt{2r}}{2r}-18c'(r_N)\,(r+r_N)^{1/2}\sqrt{r_N}\right]\sqrt{r_N}}{6\sqrt{(r+r_N)}\,[2r_N\,(r+r_N)\,c''(r_N)+(r+4r_N)]}\\ &= \frac{\sigma_w\sqrt{2rr_N}\,(r_N-2r)}{12r\,(r+r_N)^{3/2}\,[2r_N\,(r+r_N)\,c''(r_N)+(r+4r_N)]}.\end{aligned}$$

This means that the optimal level of signal precision, r_N, is increasing in σ_w. Its dependence on r is non-monotonic. If $r_N<2r$ a solution to (1.5) is locally decreasing, while for $r_N>2r$ it is locally increasing.

To find $\partial\Pi/\partial r$ and $\partial\Pi/\partial\sigma_w$ at the optimum we use the envelope theorem to get

$$\begin{aligned}\frac{\partial \Pi_A}{\partial \sigma_w} &= \frac{\partial \Pi_N}{\partial \sigma_w}=\frac{\sqrt{r_N}}{3\sqrt{2rR_N}}>0,\\ \frac{\partial \Pi_A}{\partial r} &= \frac{\partial \Pi_N}{\partial r}=-\frac{\omega\sqrt{r_N}\,(2r+r_N)}{6\sqrt{2}\,(r\,(r+r_N))^{3/2}}<0.\end{aligned}$$

As we could expect, the traders' profits are increasing in the variance of noise traders' demand and variance of the security's payoff (inverse of r).

Now, the choice of actions by both traders becomes clear. They choose the actions that would maximize their profits given the consequences that these actions make for other players. In the following proposition we find the outcome of the game, when trader A makes an announcement about innovation and trader N acquires a signal S_N.

Proposition 1.6. *If the following two conditions are satisfied:*

1. *There exists positive signal precision r_N^* such that*

$$c_N\left(r_N^*\right) < \sigma_w \frac{1}{6}\sqrt{\frac{2r_N^*}{rR_N^*}}$$

and for every r_N

$$c_N\left(r_N\right) > \sigma_w \frac{q}{6}\sqrt{\frac{2r_N}{rR_N}}$$

where $R_N = r + r_N$, and

2. *for every r_A*

$$c_A(r_A) > \sigma_w\left(\frac{1}{2}\sqrt{\frac{r_A}{rR_A}} - \frac{1}{6}\sqrt{\frac{2r_N^*}{rR_N^*}}\right)$$

then the pair of strategies $T_A = \left\{announce,\ \overline{S_A}\right\}$ and $T_N = \{S_N\}$ is the Pareto optimal Nash equilibrium outcome.

Proof. *The proof is in Appendix II. First we identify the equilibria and then check that the information announcement equilibrium is Pareto optimal.*

2. The Model with Heterogeneous Agents

Agents

There are three types of agents — a competitive risk-neutral market maker, a risk-averse trader A and n risk-neutral traders N_i with $1 \leq i \leq n$, where n is exogenously given and can be equal to 1. Trader A has exponential utility with risk-aversion coefficient a, that is, he has a CARA utility function. Both types of traders have zero initial endowment in the riskless asset and endowments $I_{j,0}$, $j = (A; N_i\}$, $1 \leq i \leq n$, in the risky asset. These endowments are initially optimal given each trader's ex ante information about the risky asset. The traders can borrow and lend at the riskless rate, which is normalized to zero. During the trading period the traders can submit demand schedules (limit orders). The market maker observes the combined value of a all orders. He also knows the number of traders, who submitted the orders, but cannot observe the individual orders. This setup is standard for the Kyle-style models. The market maker sets the price equal to the conditional expectation of the security's value.

Traders have access to a costly information production technology, which allows them to receive the signal, S_j; $j = (A; N_i\}$, $1 \leq i \leq n$, about the security's true value. For analytical tractability we assume that if both types of traders receive signals, then the signal observed by trader A is always a noisier version of statistic of the signals observed by all risk neutral traders.

All traders maximize their utility functions by making information acquisition decisions and by choosing the orders they submit. They act strategically, taking into account the effect of their decision on the actions of other traders and the market maker's pricing decision.

Information Structure and the Timeline

As before, if an innovation takes place, trader A observes a perfect binary signal φ. Trader A also knows that as a result of this innovation he will need to change his holding of the risky security, that is, he will experience a random liquidity (endowment) shock[1], I, $I \sim N(0; \sigma_I^2)$; $\sigma_I^2 = 1/r_I$.

At time t_0, trader A learns about the innovation, but does not observe the value of his corresponding liquidity shock, I, itself. The value of the endowment shock is not correlated with the new value of the risky security, V, but the liquidity shock happens if and only if there was an innovation shock.

We call the period when endowment and information shocks occur t_0. At t_0 trader A can make an announcement (truthful or not) whether the innovation has occurred or not to some traders.

If a trader decides to trade, he informs the market maker about his intention at the beginning of period t_1. By the end of t_1 the market maker and those traders, who have decided to trade, know the total number of participating traders, m. There is no entry fee or any other transaction fee that participating traders have to pay.

During period t_1 trader A and traders N_i have access to information production technologies that can produce signals, S_j, $j = \{A, N_i\}$. These signals are costly with their cost $c_j(r_j)$ being dependent on the trader's type and on the signal precision, r_j. Here we allow traders A and N to have different cost functions and so, in general $c_A(r) \neq c_N(r)$. We also assume that $c_j(r)$ are increasing functions and that their exact form are common knowledge. At this point, we do not make any additional assumptions concerning the specific shape of the cost functions. They may be continuous or they may be a discrete-valued functions. In the latter case the traders have simply to choose a pair $(r_j; c(r_j))$ from a given menu.

If no innovation has occurred, then $S_j = \mu$ for any trader who invests in a precise signal. In this case the precise signal is redundant for trader A, because he already knows that $V = \mu$. If an innovation did occur, then $S_j = V + \varepsilon_j$, where ε_j is Gaussian noise, uncorrelated with the value of the innovation, V. That is, $\varepsilon_j \sim N\left(0;\ \sigma_j^2\right)$; $\sigma_j^2 = \frac{1}{r_j}$. $j = \{A;\ N_1, \ldots,\ N_n\}$.

Assumption 3. *For analytical tractability we assume that if $r_A \leq \Sigma_j r_{N,j}$ then the signal observed by trader A is always a noisier version of a summary statistic of all the signals observed by traders N, $\varepsilon_A = \frac{1}{n}\Sigma_j \varepsilon_{N,j} + \zeta$, where ζ is a normally distributed random variable independent of $\Sigma_j \varepsilon_{N,,i}$.*

This assumption does not change the results, it simply makes the computations analytically more tractable — whenever trader A's signal is less precise than the summary statistic of all signals observed by traders N, trader A can ignore his own signal if he can observe the summary statistic (combined signal).

At the end of period t_1 trader A also observes the exact value of his endowment shock, I.

The decision tree when there are only two players — trader A and one trader N — is shown on Fig 1. For the case in which the innovation takes place, we can summarize traders' actions in the following table:

t_0	t_1 (signal acquisition)	t_2 (trading)	t_3
A suffers I; A decides on announcement;	1) A and N_i make trading decisions 2) the number of potential traders becomes known 3) traders acquire signals	traders submit demand schedules	payoff V

The players' information sets, Φ_A, Φ_N and Φ_{MM} have the following dynamics:

t_0	t_1 (signal acquisition)	t_2 (trading)	t_3
$\Phi_A = \{\varphi\}$	$\begin{pmatrix} N_i \text{ acquires } S_{N,i} \\ \Phi_A = \{I; m\} \\ \Phi_N = \{S_N; m\} \\ \Phi_{MM} = \{m\} \end{pmatrix}$ or $\begin{pmatrix} A \text{ acquires } S_A \\ \Phi_A = \{I; m; S_A\} \end{pmatrix}$ or $\begin{pmatrix} A \text{ acquires } S_A \\ N_i \text{ acquires } S_{N,i} \\ \Phi_A = \{I; m; S_A\} \\ \Phi_N = \{m; S_N\} \\ \Phi_{MM} = \{m\} \end{pmatrix}$	$\begin{pmatrix} A \text{ submits } x(p; I) \\ N_i \text{ submits } z_i(p; S_{N,i}) \\ \Phi_A = \{I; m; p\} \\ \Phi_{N_i} = \{S_{N,i}; m; p\} \\ \Phi_{MM} = \{m; y\} \end{pmatrix}$ or $\begin{pmatrix} \text{A submits } x(p; I; S_A) \\ \Phi_A = \{I; S_A\} \end{pmatrix}$	V

Trader A wants to maximize the certainty equivalent of his wealth. His motivation to trade is caused by both information and risk-sharing reasons.

Essentially, trader A faces a choice between the following options: 1) he can acquire a signal, $S_A = V + \varepsilon_A$, $\varepsilon_A \sim N\left(0; \sigma_A^2\right); \sigma_A^2 = 1/r_A$ and trade on this information, 2) he can make an announcement about the innovation, thus stimulating information production by traders N_i and then submit the demand schedule (limit order) incorporating traders' N_i information; or 3) without acquiring information and announcing he can trade away all his endowment, I.

We will show that for the information costs that are high enough, the third option can be the equilibrium outcome. In fact, whenever this outcome is possible, it is trader A's preferred choice. However, for a wide range of information costs, the pair $(r_A = 0; p = \mu)$ cannot be a Nash equilibrium of the subgame between the market maker and trader A, because if the market maker sets price $p = \mu$ (the best response to the uninformed trade with the signal precision $r_A = 0$), then trader A has an incentive to deviate by acquiring a signal with $r_A > 0$. The market maker rationally anticipates this and sets the price accordingly. Therefore, when the information cost is low, trader A acquires the signal himself. But as the cost gets bigger, trader A becomes less willing to acquire information.

There is a range of price functions, $c_A(r_A)$, for which in a pure linear Nash equilibrium trader A cannot conceal or falsify his knowledge of the innovation. Precise information is too costly for trader A to trade against the upward sloping pricing function, but this cost is not high enough to prevent trader A from acquiring it if $p = \mu$. Trader A cannot credibly commit not to acquire information and the market maker anticipates that.

We find a range of parameters for which there is only one pure strategy linear Nash equilibrium. An interesting feature of this equilibrium is that trader A makes an announcement about the innovation in order to trigger information production by trader(s) N. One might argue that this is merely a mathematical result, but we believe that it has a deeper meaning. This linear equilibrium means less uncertainty for the risk-averse trader A than a mixed equilibrium with non-linear pricing. Trader A also saves on the signal acquisition costs. The trade-off is that the market may become less liquid when the market maker increases the slope of the supply curve knowing that trader N is participating.

For other parameter values multiple equilibria are possible. As we have said, whenever trading away the endowment without making an announcement and acquiring information is part of the equilibrium, trader A will prefer this outcome to all others. When trading informed without announcement is another equilibrium, then the outcome depends on trader A's expected utility. There are sets of parameters, when trader A decides to make an announcement about the innovation, because his expected utility is greater than if he acquires information himself and trades alone.

3. One Risk-Averse and One Risk-Neutral Trader ($n = 1$)

As we have mentioned in the introduction, the main purpose of this paper is to examine the equilibrium in which heterogeneous agents share their private information. In this section there is one risk-averse trader, A, and one risk-neutral trader, N. After suffering the endowment shock, I, which signals that an innovation has taken place, trader A has to choose the optimal strategy. Being risk-averse, he wants to maximize his expected utility even at the cost of possible financial losses. Therefore, his decision whether to acquire information himself or to announce that the innovation has taken place depends on particular characteristics of the information production technologies, namely the signal precision and the cost functions that are available to each trader, on the security's volatility and on trader A's risk-aversion, a.

First, as a benchmark, we find trader A's utility if trader N does not trade. In this case the model becomes very similar to Glosten (1989). The signal acquisition decision is in the spirit of Admati and Pfleiderer (1986).

3.1. Case when trader A trades alone

Instead of announcing the innovation, trader A can acquire information himself or even trade uninformed. In fact, if he could credibly commit to staying uninformed, he would be better off by trading out all his endowment, that is by submitting order $x = I$. In this case the market maker would set price $p = \mu$ and trader's A expected utility would become simply

$$-\exp\{-a\mu I\}$$

The problem for trader A is that he cannot restrain himself from acquiring information if he knows that the price will remain constant $p = \mu$. He would choose to acquire information to maximize his expected utility. Knowing that, the market maker would set the price $p = \mu + \lambda_A x$ equal to the conditional expectation of the security's value V. The linear Nash equilibrium of a subgame between the market maker and trader A is a pair of strategies chosen by the market maker and by trader A. The market chooses the pricing function pricing function, $p = \mathrm{E}\{V|x\} = \mu + \lambda_A$, and trader A chooses signal precision,

r_A, and demand schedule x such that

$$\begin{aligned} r_A &= \arg\max_{r_A} \mathrm{E}\left\{U_A(\cdot)|\lambda_A\right\} \\ x &= \arg\max_{x} \mathrm{E}\left\{U_A(\cdot)|\lambda_A; S_A\right\} \end{aligned}$$

where

$$\lambda_A = \arg\left(\mathrm{E}\left\{V|x\right\} = \mu + \lambda_A x\right)$$

We have the following Proposition:

Proposition 3.1. *Let $c_A(r_A)$ be the cost paid by trader A for acquiring signal $S_A = V + \varepsilon_A; \varepsilon_A \sim N\left(0; \sigma_A^2\right); \sigma_A^2 = 1/r_A$.*

1. *if for every r_A*
$$c(r_A) > \frac{1}{2a} \ln\left(1 + \frac{r_A}{r}\right)$$
then decision to remain uninformed always dominates the information acquisition decision. If trader A remains uninformed, does not announce and trades out all his endowment shock, he gets expected utility
$$-\exp\left\{-a\mu I\right\}$$

2. *if there exists r_A such that*
$$0 \leq c_A(r_A) \leq B < \frac{1}{2a} \ln\left(1 + \frac{r_A}{r}\right)$$
where
$$B = \frac{1}{2a} \ln\left[1 + \frac{r_A\left(a^2 r - r_A R_A r_I\right)}{r\left(a^2 r + r_A R_A r_I\right)}\right]$$
and $R_A = r + r_A$, then the decision to acquire information is the dominant decision.

If trader A acquires information and trades alone, his expected utility is

$$U_A = -\sqrt{\frac{r\left(r_A R_A r_I + a^2 r\right)}{R_A\left(a^2\left(r-2r_A\right)+r_A(r-r_A)r_I\right)}} \times \times \exp\left\{\frac{a^2\mu^2\left(a^2 r + r_A(r-r_A)r_I\right)}{2r_I\left(a^2\left(r-2r_A\right)+r_A(r-r_A)r_I\right)} + ac_A(r_A)\right\}. \tag{3.1}$$

The market maker sets the price equal to

$$p = \mu + \lambda_A x$$

where x is the order submitted by trader A

$$x = \frac{r_A}{2\lambda_A R_A + a}(S_A - \mu) - \frac{a}{2\lambda_A R_A + a} I;$$

and λ is the market depth

$$\lambda_A = \frac{a r_A}{\frac{a^2 r}{r_I} - r_A R_A}$$

where $R_A = r + r_A$.

Proof. *The proof is in Appendix II*

Corollary 3.2. *In order to keep λ_A finite ($\lambda_A < \infty$) r_A must satisfy*

$$0 \le r_A < r_A^c$$

where

$$r_A^c = \frac{-r + \sqrt{r^2 + 4r\frac{a^2}{r_I}}}{2} < \frac{a^2}{r_I}$$

Proof. *The proof is straightforward. It follows from the condition $\lambda_A \geq 0$.*

Proposition 3.1 establishes the result that if the information acquisition cost is high enough, then trader A prefers to remain uninformed even with $p = \mu$. For the low information acquisition cost, it is possible that trader A decides to become informed. However, the low information acquisition cost itself does not necessarily guarantee that the decision to become informed is part of the Nash equilibrium. For a wide range of parameters, a pure Nash equilibrium does not exist even for zero cost! Notice also that when the strategy to become informed is part of the Nash equilibrium, it does not mean that trader A's utility is higher than in the case of the equilibrium in which he stays uninformed.

We have two possible situations, when there is no pure strategy equilibrium. First, if $r_A > r_A^c$, then the market completely breaks down and there is no equilibrium at all. This situation is known in the literature (see, e.g., Glosten (1989)). Figure 2 shows trader A's expected utility and the market depth parameter (price sensitivity), λ_A, as functions of signal precision, r_A. For this and the following figures we use the following parameter values: $c(r_A) = 0$, $r = 3$, $r_I = 2$, $\mu = 1$, and $a = 1.5$. Notice, that even with zero signal acquisition cost, trader A's utility is strictly decreasing as the function of r_A.

The critical value for r_A is $r_A^c = .8717$. This is the value at which λ_A would become infinite. This is the principal difference with the classical Kyle model with noise traders. The informed trader in the Kyle model can observe a perfect signal and the market does not break down thanks to the presence of noise traders. Since the risk averse trader trades for both information and risk sharing reasons, the higher the information precision, the lower is the risk sharing component of his trades, and, therefore, the bigger price sensitivity to the signal precision. As Bhattacharya and Spiegel (1991) point out, the market breaks down when the informational motive for trade of the insider outweights her hedging motive. The market collapse condition extends not only to the linear pricing function, but to the whole class of feasible nonlinear strategy functions.

However, there is a second scenario when no pure strategy equilibrium of the sub-game between trader A and the market maker exists. This pertains when trader A cannot credibly commit to remain uninformed, because the cost of information is not high enough. At the same time, if trader A must trade against the upward sloping pricing schedule, the information cost is too high.

Corollary 3.3. *No pure strategy equilibrium of a sub-game between trader A and the* *market maker exists for*

$$B < c(r_A) < \frac{1}{2a} \ln\left(1 + \frac{r_A}{r}\right)$$

Figure 3 shows two regions with possible pure strategy equilibria for the case when trader A does not make an announcement about the innovation. When the information cost is high for all r_A (the top left corner), trader A remains uninformed and the market maker anticipates this. The bottom region corresponds to the equilibrium outcome where trader A becomes informed, because the cost is low. If trader A trades alone, then no pure strategy equilibrium is possible in the region between these two areas. We cannot rule out the existence of a mixed strategy equilibrium. In such an equilibrium, trader A randomly decides whether to acquire information or not and the market maker takes the probability of informed trade into account, when making the pricing decision. Such an equilibrium cannot be linear and its study is beyond the scope of this paper.

3.2. Linear equilibrium for traders A and N

Our goal is to find the pure Nash linear equilibria for the entire game, when trader N can also participate in the trade. The game tree is depicted on Figure 1. The dotted lines connect the nodes which belong to the same information set for a given trader. If trader A conceals his information about the innovation, then at time t_0 trader N does not know whether the innovation has occurred or not. If innovation has taken place and trader A conceals this information, then at time t_1 he does not know whether trader N has acquired a signal or not. When no innovation takes place, the trader A's utility is equal to -1 ($U_A = -\exp(0)$).

Before the trading takes place, trader A makes two moves. In his first move he chooses whether to make an announcement or to conceal his information about the innovation. His announcement is not required to be truthful — when announcing, he can make a false statement, claiming that the innovation has taken place, when it has not and vice versa. In the second move trader A chooses whether or not to acquire a signal. Trader N makes only one move, when he makes an information acquisition decision. We will show that both players always trade and therefore we do not include decision to trade as a move of the game.

Formally we denote trader A's strategy as $T_A = (T_{A,1}; T_{A,2})$ where the possible moves are $T_{A,1} = \{$*announce*, *conceal*, *falsify*$\}$ and $T_{A,2} = \{S_A; \overline{S_A}\}$. For trader N, $T_N = \{S_N; \overline{S_N}\}$. Here S_j means that trader j decides to acquire a signal and $\overline{S_j}$ means that trader j does not acquire a signal.

We will find conditions, under which in the linear equilibrium trader A truthfully makes an announcement about the innovation and remains uninformed, while trader N acquires a signal, S_N, incurring the acquisition cost. Depending on the information cost function and other parameter values, this can be either unique pure strategy equilibrium or one of several possible equilibria. Even when other equilibria exist, in which trader A conceals or falsifies information, it is possible that trader A prefers the information announcement equilibrium, because it gives him a higher expected utility.

His choice to make an announcement can be explained in the following way. When there are two traders and both of them submit demand schedules (limit orders), then at the time of trade execution, t_3, both traders have the same information set. Therefore, trader N correctly infers the noise component of trader A's demand schedule and partially compensates this noise component in order to smoothen the price reaction. Trader A expects to have a financial loss, but his expected utility can go up, because of ability to share risk with more agents. Similar result takes place in Admati and Pfleiderer (1988), where a risk-averse agent prefers to sell information to share the risk.

When we take into account the information acquisition cost, the incentive to disclose information instead of acquiring a more precise costly signal can become even stronger. First, we assume that traders have access to different information production technologies, therefore it would possibly be more expensive for trader A to acquire the same signal. Second, the cost paid for the signal is deterministic, while the profits, although positive, are stochastic. This factor is also detrimental to trader A's decision to acquire information. As we have shown in Proposition 3.1, a pure strategy equilibrium with trader A acquiring information does not always exist.

In the following proposition we find the outcome of the game, when trader A makes an announcement about innovation and trader N acquires a signal S_N

Proposition 3.4. *If trader A makes an announcement about the innovation and ex-*

periences endowment shock I and trader N acquires signal S_N, $S_N = V + \varepsilon_N$, $\varepsilon_N \sim N\left(0; \sigma_N^2\right); \sigma_N^2 = 1/r_N$ and

$$a^2 r R_N - 2 r_N r_I R_N^2 > 0 \tag{3.2}$$

then there exists a unique linear equilibrium with pricing schedule

$$p(y) = \mu + \lambda_N y,$$

trader A's demand schedule

$$x = \frac{R_N}{a + 2\lambda_N R_N}(p - \mu) - \frac{a}{(a + 2\lambda_N R_N)} I \tag{3.3}$$

and trader N's demand schedule

$$z = \frac{r_N}{\lambda_N R_N}(S_N - \mu) - \frac{p - \mu}{\lambda_N} \tag{3.4}$$

where $R_N = r + r_N$ and

$$\lambda_N = \frac{3 a r_N r_I + a\sqrt{(r_N r_I)^2 + 4a^2 \frac{r r_N r_I}{R_N}}}{2\left(a^2 r R_N - 2 r_N r_I R_N^2\right)} \tag{3.5}$$

Proof. *The proof is in Appendix II*

Corollary 3.5. *The equilibrium exists only if r_N is smaller than the critical value r_N^c*

$$0 \leq r_N < r_N^c = \frac{-r + \sqrt{r^2 + 2\frac{a^2 r}{r_I}}}{2} < \frac{a^2}{2 r_I}$$

Proof. *The proof follows from the condition $\lambda_N \geq 0$.*

Notice that λ_N is a much steeper function of r_N than λ_A is a function of r_A and $r_N^c < r_A^c$. This means that the market with two informed traders breaks down for signal precision levels that are lower than for the market with only trader A. This is because for the same precision level of the signal, the combined order of traders A and N contains relatively less noise than the order coming from trader A only. As we have mentioned, this happens because trader N partially compensates for the noise component of trader A's order, as we can see by comparing expressions (7.10) and (7.11).

The following corollary establishes conditions when trader N prefers to become informed.

Corollary 3.6. *If trader A makes an announcement about the innovation and there exists r_N such that*

$$c_N(r_N) < \frac{1}{\lambda_N}\left[\frac{r_N\left(a+\lambda_N R_N\right)^2}{rR_N\left(2a+3\lambda_N R_N\right)^2}\right]$$

where λ_N is determined by expression (3.5), then trader N prefers to acquire signal S_N and his expected profit is

$$\Pi_N = \frac{1}{\lambda_N}\left[\frac{r_N\left(a+\lambda R_N\right)^2}{rR_N\left(2a+3\lambda R_N\right)^2}+\frac{\left(a\lambda_N\right)^2}{r_I\left(2a+3\lambda_N R_N\right)^2}\right]-c_N(r_N) \tag{3.6}$$

Proof. *The proof is in Appendix II, but its idea is straightforward. We compare trader N's expected profits when he trades remaining uninformed with his expected profits when he becomes informed.*

Figure 4 in Appendix III shows how λ_N and Π_N change with r_N. We can see that λ_N is increasing with r_N while Π_N initially increases, then reaches its maximum at $r_N = .275$ and starts decreasing. Its decrease for high values of r_N can be explained by the high value of λ_N and the relatively low level of "noise trading" determined by volatility of the endowment shock I. The critical value, $r_N^c = .4843$, which is smaller than the value r_A^c.

Now we are ready to identify conditions, sufficient for the pair of strategies $\mathbf{T}=((announce, \overline{S_A}); S_N)$ played by traders A and N to constitute the linear Nash equilibrium. The following proposition is the main result of this section, because it establishes the range of parameters for which cooperation in information production is the **unique** linear Nash equilibrium even when other, nonlinear, equilibria exist.

Proposition 3.7. *If the following conditions hold:*

1. *there exists signal precision, r_N, such that*

$$c_N(r_N) < \frac{1}{\lambda_N}\left[\frac{r_N\left(a+\lambda_N R_N\right)^2}{rR_N\left(2a+3\lambda_N R_N\right)^2}\right]$$

and

$$0 < \left(a^2 rR_N - 2r_N r_I R_N^2\right)$$

where

$$\lambda_N = \frac{3ar_N r_I + a\sqrt{\left(r_N r_I\right)^2 + 4a^2\frac{rr_N r_I}{R_N}}}{2\left(a^2 rR_N - 2r_N r_I R_N^2\right)}$$

2. *for every r_N*

$$c_N(r_N) > q\frac{1}{\lambda_N}\left[\frac{r_N\left(a+\lambda R_N\right)^2}{rR_N\left(2a+3\lambda R_N\right)^2}\right]$$

and

3. *for every r_A*

$$B \leq c_A(r_A) < \frac{1}{2a}\ln\left(1+\frac{r_A}{r}\right)$$

where

$$B = \frac{1}{2a}\ln\left[1+\frac{r_A\left(a^2 r - r_A R_A r_I\right)}{r\left(a^2 r + r_A R_A r_I\right)}\right]$$

then the pair of strategies $T_A = (announce, \overline{S_A})$ and $T_N = (S_N)$ is the only pure linear Nash equilibrium of this game.

Proof. *The proof is in Appendix II. First we verify that $T_A = (announce, \overline{S_A})$ and $T_N = (S_N)$ is indeed a linear Nash equilibrium and then we check the other candidates for the equilibrium.*

Conditions 1 and 2 of Proposition 3.7 state that for trader N it is profitable to acquire information only if he knows for sure that the innovation has taken place. Condition 3 is necessary to have only one linear equilibrium. If it holds, then the conditions of

Proposition 3.1 are violated and concealing or falsifying cannot be a part of the pure equilibrium strategy for trader A.

4. Discussion and Conclusion

The established result gives us the sufficient conditions for traders A and N to get involved in the information sharing game. Whenever all conditions of Proposition 3.7 hold, information sharing is the only linear equilibrium outcome of the game. For trader A such an outcome resembles more a marriage of convenience than a marriage of love — he chooses information sharing as the lessor of two evils. Therefore, it is interesting to look at the cases when condition 3 on the cost function of trader A is relaxed, that is, when condition 1 or condition 2 of Proposition 3.1 holds and another pure strategy equilibrium is possible. Then we have to compare trader A's expected utility as an outcome of the information sharing game with his expected utility without information sharing. With information sharing, trader A's utility is

$$\begin{aligned} U_A &= \mathrm{E}\left[\mathrm{E}\left[-\exp\left\{-a\left(V\left(x+I\right)-p(y)x\right)\right\}\middle|\, S_N;I\right]\right] \\ &= \mathrm{E}\left[\mathrm{E}\left[-\exp\left\{-a\left(V\left(\frac{r_N\left(S_N-\mu\right)}{\left(3\lambda_N R_N+2a\right)}+\frac{3\lambda_N R_N I}{3\lambda_N R_N+2a}\right)-\times\right.\right.\right.\right. \\ &\quad -\left(\mu+\lambda\left(\frac{r_N\left(2\lambda_N R_N+a\right)\left(S_N-\mu\right)}{\lambda_N R_N\left(3\lambda_N R_N+2a\right)}-\frac{aI}{3\lambda_N R_N+2a}\right)\right)\times \\ &\quad \left.\left.\left.\left.\times\left(\frac{r_N\left(S_N-\mu\right)}{\left(3\lambda_N R_N+2a\right)}-\frac{2aI}{3\lambda_N R_N+2a}\right)\right)\right\}\right|\, S_N;I\right]\right] \end{aligned}$$

Taking expectations with respect to first V and then I and using (6.8) we get

$$\begin{aligned} U_A &= -\frac{\left(3\lambda_N R_N+2a\right)\sqrt{r}}{\sqrt{r\left(3\lambda_N R_N+2a\right)^2+ar_N\left(2\lambda_N R_N+a\right)}}\,\mathrm{E}\left[\exp\left\{-a\left(\mu I-\frac{aI^2}{2\left(3\lambda_N R_N+2a\right)^2}\times\right.\right.\right. \\ &\quad \left.\left.\left.\times\left[\lambda_N\left(9\lambda_N R_N+4a\right)+\frac{r_N\left(\left(3\lambda_N R_N+2a\right)^2-2a\left(2\lambda_N R_N+a\right)\right)^2}{\left(r\left(3\lambda_N R_N+2a\right)^2+ar_N\left(2\lambda_N R_N+a\right)\right)R_N}\right]\right)\right\}\right] \\ &= -\frac{\left(3\lambda_N R_N+2a\right)\sqrt{r}}{\sqrt{r\left(3\lambda_N R_N+2a\right)^2+ar_N\left(2\lambda_N R_N+a\right)}}\frac{1}{\sqrt{1-\frac{2A}{r_I}}}\exp\left\{\frac{a^2\mu^2}{2\left(r_I-2A\right)}\right\} \end{aligned}$$

where

$$A = \frac{a}{2\left(3\lambda_N R_N + 2a\right)^2}\left[\lambda_N\left(9\lambda_N R_N + 4a\right) + \frac{r_N\left(\left(3\lambda_N R_N + 2a\right)^2 - 2a\left(2\lambda_N R_N + a\right)\right)^2}{\left(r\left(3\lambda_N R_N + 2a\right)^2 + ar_N\left(2\lambda_N R_N + a\right)\right)R_N}\right]$$

Figure 5 shows trader A's utilities as functions of the signal precision for different possible outcomes of the game. The information acquisition cost is zero, $c = 0$, and other parameters have the following values: $r = 3$, $r_I = 2$, $\mu = 1$, $a = 1.5$. The solid downward sloping line is trader A's utility when he makes an announcement about the innovation and trader N acquires the signal. It is a function of information precision r_N chosen by trader N. The dotted downward sloping line is trader A's expected utility when he becomes informed and trades alone in equilibrium. The dashed horizontal line is his expected utility from trading uninformed, when sunshine trading[2] is possible ($\lambda = 0$). The upward sloping solid line shows trader A's expected utility when he trades informed, while the market maker keeps $\lambda = 0$. This line does not represent an equilibrium outcome, it merely shows trader A's incentive to become informed. Information acquisition costs are zero. Stars mark the equilibrium point when trader N's profit reaches the maximum.

We can see, that whenever the no-information equilibrium is possible, trader A prefers this outcome, i.e., he remains uninformed. Normally, this happens when signal acquisition costs are high. More interesting is the case when an information acquisition equilibrium without announcement exists. Nevertheless, for a large set of parameters (including our numerical example), trader A prefers to make an announcement about the innovation rather than to acquire information himself. Finally, for the intermediate region of Figure 3, information announcement is the only pure strategy linear equilibrium that exists.

4.1. One Risk-Averse and Many Risk-Neutral Traders ($n > 1$)

One possible extension of the paper would be the case with multiple risk-neutral traders. Potentially, trader A has larger benefits if his announcement reaches a higher number of risk neutral traders. This happens for two reasons. First, because risk neutral traders compete with each other, thus making the security market more liquid. Second, because trader A's order is the only source of uncertainty, he has an informational advantage over other traders — from the price he can perfectly infer the combined signal, which is the summary statistic of traders' N_i individual signals, while traders N_i observe this combined signal with an error equal to the value of the endowment shock, I. Therefore, we have a paradoxical situation, when the least privately informed agent gets the finest signal from the observed price. This situation is somewhat similar to Brunnermeier (1998), although in his model a trader who gets the noisiest signal does not become the best informed, but is simply able to price manipulate.

Notice also that as the number of traders becomes bigger, the market maker's information set becomes almost equal to that of traders N_is', thus driving the latters' profits to zero and giving them less and less incentive to acquire a signal. On the other hand, as the combined signal becomes more precise, trader A trades more and more for information reasons and less and less for risk-sharing reason. Therefore a point can be reached, when there is no uncertainty and therefore on incentive to acquire the signal.

4.2. Conclusion

Taking the market microstructure model as an example, we have shown how an agent can increase his expected utility by disclosing his private information.

A person, who has access to a better (more precise) information production technology is not necessarily the one who gets the most benefit from it. On the contrary, in our model, as in Brunnermeier (1998), sometimes the biggest winner is a person with the worst information production technology, because his imprecise signal serves as a source of noise that allows trades to go through.

Notice also that overt public announcement is not always necessary to make the information disclosure, because the latter can be a by-product of other actions. For example, in a multi-period setup with more than one round of trading, the price change in the first round gives a signal about the innovation. In such a case price can serve as the only communication channel between the traders.

We expect to observe increases in trading volume and stock price volatility **after** "unexpected" public announcements and information leaks. This is because these public announcements can be made with the aim of starting a new information generation process. For example, this could explain observed price volatility clustering.

In industries where information production technology is more expensive and uncertainty is very high, one might expect to observe more information sharing than in industries with low degree of uncertainty and low information costs.

5. Notation

V — value of the risky security, $\mathrm{E}[V] = \mu$

q — probability of innovation

$r = \frac{1}{\sigma^2}$ — inverse of the variance of innovation

φ — perfect binary signal about the innovation

S_j — signal about the value of innovation observed by trader $j = \{A, N\}$, $S_j = V + \varepsilon_j, \varepsilon_j \sim N(0; \sigma_j^2)$

$r_j = \frac{1}{\sigma_j^2}$ — signal precision

$c_j(r_j)$ — signal cost

$R_j = r + r_j$

$\Omega = r + r_A + r_N$

m — the number of participating traders

z — trader N's order

x — trader A's order

w — noise traders' order

y — net order observed by MM, $y = x + z + w$

$p(y)$ — competitive price set by MM

λ — Kyle's lambda (linear coefficient in the pricing function)

Π_j — expected profit of trader j

a — coefficient of risk aversion when trader A is a CARA trader

I — endowment shock experienced by trader A. $I \sim N(0; \sigma_I^2)$; $\sigma_I^2 = 1/r_I$.

6. Appendix I

If

$$\begin{aligned} z &= x + y \\ f_x &\sim N(\mu_x; \sigma_x^2) \\ f_y &\sim N(\mu_y; \sigma_y^2) \end{aligned}$$

then

$$\mathrm{E}\left[x|z\right] = \frac{\mu_x \sigma_y^2 + \mu_y \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \tag{6.1}$$

or, more general, if $\mathbf{x}$ and $\mathbf{y}$ are $n \times 1$ and $m \times 1$ jointly normal vectors, then we have the following formula (in Greene (1993) on p.76)

$$\mathrm{E}(\mathbf{x}|\mathbf{y}) = \mathrm{E}\left(\mathbf{x}\right) + \mathrm{Cov}(\mathbf{x};\mathbf{y})\,\mathrm{Var}(\mathbf{y})^{-1}(\mathbf{y} - \mathrm{E}(\mathbf{y})) \tag{6.2}$$

$$\mathrm{Var}(\mathbf{x}|\mathbf{y}) = \mathrm{Var}\left(\mathbf{x}\right) - \mathrm{Cov}(\mathbf{x};\mathbf{y})\,\mathrm{Var}(\mathbf{y})^{-1}\,\mathrm{Cov}(\mathbf{x};\mathbf{y})^{\mathbf{T}} \tag{6.3}$$

$$\mathrm{Cov}(\mathbf{x},\mathbf{z}|\mathbf{y}) = \mathrm{Cov}\left(\mathbf{x},\mathbf{z}\right) - \mathrm{Cov}(\mathbf{x};\mathbf{y})\,\mathrm{Var}(\mathbf{y})^{-1}\,\mathrm{Cov}(\mathbf{y};\mathbf{z}) \tag{6.4}$$

where $\mathrm{Cov}(\mathbf{x};\mathbf{y})$ is $n \times m$ covariance matrix of $\mathbf{x}$ and $\mathbf{y}$

Some other useful formulas:

$$\int \exp\{ax\} f_x dx = \exp\left\{\mu_x a + \frac{1}{2} a^2 \sigma_x^2\right\} \tag{6.5}$$

$$\int \exp\{ax^2\} f_x dx = \frac{1}{\sqrt{1 - 2a\sigma_x^2}} \exp\left\{\frac{a\mu^2}{1 - 2s\sigma_x^2}\right\} \tag{6.6}$$

$$
\begin{aligned}
& \int (ax+b) \exp\left\{-(cx+d)^2\right\} f_x dx \\
= \quad & \left[a \frac{\mu_x - 2cd\sigma_x^2}{\left(1+2c^2\sigma_x^2\right)^{3/2}} + \frac{b}{\sqrt{1+2c^2\sigma_x^2}} \right] \exp\left\{ -\frac{(\mu_x c + d)^2}{1+2c^2\sigma_x^2} \right\}
\end{aligned} \tag{6.7}
$$

$$
\int \exp\left\{ax^2+bx+c\right\} f_x dx = \frac{1}{\sqrt{1-2a\sigma_x^2}} \exp\left\{ c + \frac{2\left(b\mu_x + a\mu_x^2\right) + \sigma_x^2 b^2}{2\left(1-2a\sigma_x^2\right)} \right\} \tag{6.8}
$$

7. Appendix II

Proof. Proposition 1.1: When trading alone, trader A maximizes his expected profit by choosing the limit order x

$$\max_x \mathrm{E}\left[(V-p)\,x|\,S_A^*;p,w\right]$$

Assuming linear pricing rule $p=\mu+\lambda(x+w)$ and using Expression (6.1) we get

$$x=\frac{\mathrm{E}\left[V|S_A^*\right]-\mu}{2\lambda}-\frac{w}{2}=\left(S_A^*-\mu\right)\frac{r_A^*}{2\lambda R_A^*}-\frac{w}{2} \tag{7.1}$$

where $R_A^*=r+r_A^*$. The market maker sets the price equal to the conditional expectation of V given the combined order, $y=x+w$, that he observes. The combined order, y, has the following statistical properties

$$\mathrm{E}\left[y\right]=0, \mathrm{Var}\left[y\right]=\frac{1}{4\lambda^2}\frac{r_A^*}{rR_A^*}+\frac{\sigma_w^2}{4}, \mathrm{Cov}\left(V,y\right)=\frac{1}{2\lambda}\frac{r_A^*}{rR_A^*} \tag{7.2}$$

Using expressions (6.2), (6.3), (6.4) and (7.2)

$$p=\mathrm{E}\left[V|y\right]=\mu+\lambda y$$

where

$$\lambda=\frac{1}{\sigma_w}\sqrt{\frac{r_A^*}{rR_A^*}}. \tag{7.3}$$

Substituting (7.3) into (7.1) we get the result. □

Proof. Proposition 1.3: Both traders want to maximize their expected profits. Let us assume that

$$p=\mu+\lambda y=\mu+\alpha(S_N-\mu)+\beta w.$$

Trader A maximizes

$$\max_x \mathrm{E}\left[\left(V - p(y)\right) x \middle| p; w\right] \tag{7.4}$$

by setting

$$x = \frac{\mathrm{E}\left[\left(V\right) x \middle| p; w\right] - p}{\lambda}$$

and trader N maximizes

$$\max_z \mathrm{E}\left[\left(V - p(y)\right) z \middle| p; S_N\right] \tag{7.5}$$

by setting

$$z = \frac{\mathrm{E}\left[\left(V\right) x \middle| p; S_N\right] - p}{\lambda}.$$

Using (6.2), (6.4), (6.3) and

$$\begin{aligned}
\mathrm{Var}\left[p; S_N\right] &= \begin{bmatrix} \alpha^2 \frac{R_N}{rr_N} + \beta^2 \sigma_w^2 & \alpha \frac{R_N}{rr_N} \\ \alpha \frac{R_N}{rr_N} & \frac{R_N}{rr_N} \end{bmatrix}; \\
\mathrm{Var}\left[p; S_N\right]^{-1} &= \frac{1}{\beta^2 \sigma_w^2 \frac{R_N}{rr_N}} \begin{bmatrix} \frac{R_N}{rr_N} & -\alpha \frac{R_N}{rr_N} \\ -\alpha \frac{R_N}{rr_N} & \alpha^2 \frac{R_N}{rr_N} + \beta^2 \sigma_w^2 \end{bmatrix};
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}\left[p; w\right] &= \begin{bmatrix} \alpha^2 \frac{R_N}{rr_N} + \beta^2 \sigma_w^2 & \beta \sigma_w^2 \\ \beta \sigma_w^2 & \sigma_w^2 \end{bmatrix}; \\
\mathrm{Var}\left[p; S_N\right]^{-1} &= \frac{1}{\alpha^2 \sigma_w^2 \frac{R_N}{rr_N}} \begin{bmatrix} \sigma_w^2 & -\beta \sigma_w^2 \\ -\beta \sigma_w^2 & \alpha^2 \frac{R_N}{rr_N} + \beta^2 \sigma_w^2 \end{bmatrix}.
\end{aligned}$$

we can write

$$\begin{aligned}
\mathrm{E}\left[\left(V\right) x \middle| p; w\right] &= \mu + \frac{r_N}{\alpha R_N}\left(p - \mu\right) - \frac{\beta r_N}{\alpha R_N} w; \\
\mathrm{E}\left[\left(V\right) x \middle| p; S_N\right] &= \mu + \frac{r_N}{R_N}\left(S_N - \mu\right).
\end{aligned}$$

and then

$$x = \left(\frac{r_N}{\alpha R_N} - 1\right)\frac{(p-\mu)}{\lambda} - \frac{\beta r_N}{\lambda \alpha R_N} w,$$

$$z = \frac{r_N}{\lambda R_N}(S_N - \mu) - \frac{(p-\mu)}{\lambda},$$

$$y = x + z + w = \frac{p-\mu}{\lambda}.$$

Solving these equations together with $p = \mu + \alpha(S_N - \mu) + \beta w$ we get

$$\alpha = \frac{2r_N}{3R_N};\ \beta = \frac{\lambda}{3}.$$

Since $p = \mathrm{E}\,(V|y)$we can solve for λ using (6.2) to get

$$\lambda = \frac{1}{\sigma_w}\sqrt{\frac{2r_N}{rR_N}}$$

and the result follows. □

Proof. Proposition 1.6: The traders choose demands to maximize their expected profits given the signals that they observe. From Corollary 1.2 we know that when trader A trades alone, his profit is

$$\Pi_{A,1} = \frac{\omega}{2}\sqrt{\frac{r_A}{rR_A}} - c_A(r_A).$$

Corollary 1.5 gives us the trader A's expected profit

$$\Pi_{A,2} = \frac{1}{6}\sigma_w\sqrt{\frac{2r_N^*}{rR_N^*}}$$

and trader N's expected profit

$$\Pi_N = \frac{1}{6}\sigma_w \sqrt{\frac{2r_N^*}{rR_N^*}} - -c_N\left(r_N^*\right)$$

when both traders submit limit orders. If trader N does not know that the innovation took place, then his expected profit from trading is

$$\Pi_N = \frac{q}{6}\sigma_w \sqrt{\frac{2r_N^*}{rR_N^*}} - -c_N\left(r_N^*\right)$$

We denote by $\mathrm{E}\left\{\Pi_i\left(T_A; T_N\right)\right\}$, $i = \{A;\ N\}$ trader i's expected profit when strategies T_A and T_N are played. Then the pair of strategies $T_A = (announce,\ \overline{S_A})$ and $T_N = S_N$ is the Nash equilibrium if the following inequalities hold:

$$\begin{aligned}
\mathrm{E}\left\{\Pi_A((announce, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((conceal, \overline{S_A}); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((announce, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((falsify, \overline{S_A}); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((announce, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((conceal, S_A); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((announce, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((falsify, S_A); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((announce, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((announce, S_A); S_N)\right\} \\
\mathrm{E}\left\{\Pi_N((announce, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_N((announce, \overline{S_A}); \overline{S_N})\right\}
\end{aligned}$$

The first two expressions are equalities. Expressions 3-5 are strict inequalities, because of the information cost borne by trader A and the nature of the signal $S_A = S_N + \varepsilon_{A/N}$. When trader N does not acquire a signal and trades uninformed, he makes a zero profit. Therefore, the sixth inequality holds whenever

$$\frac{1}{6}\sigma_w \sqrt{\frac{2r_N^*}{rR_N^*}} - -c_N\left(r_N^*\right) \geq 0.$$

The pair of strategies $T_A = (conceal, \overline{S_A})$ and $T_N = S_N$ is the Nash equilibrium if the following inequalities hold:

$$\begin{aligned}
\mathrm{E}\left\{\Pi_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((announce, \overline{S_A}); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((falsify, \overline{S_A}); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((announce, S_A); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((falsify, S_A); S_N)\right\} \\
\mathrm{E}\left\{\Pi_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_A((conceal, S_A); S_N)\right\} \\
\mathrm{E}\left\{\Pi_N((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{\Pi_N((conceal, \overline{S_A}); \overline{S_N})\right\}
\end{aligned}$$

The first two expressions are equalities. Expressions 3-5 are strict inequalities, because of the information cost $c_A(r_A)$. The last one holds whenever

$$\frac{q}{6}\sigma_w\sqrt{\frac{2r_N^*}{rR_N^*}} - -c_N\left(r_N^*\right) \geq 0$$

We also have to check the pair $T_A = (\textit{falsify}, \overline{S_A})$ and $T_N = S_N$. Since trader A has no incentive to lie when the event does not actually happen, he can possibly falsify only when the innovation has taken place. The probability of $E = \{innovation\}$ when trader A says that there is no innovation is

$$\Pr\left[E|No\right] = \frac{\Pr\left[No|E\right]\Pr\left[E\right]}{\Pr\left[No|E\right]\Pr\left[E\right] + \Pr\left[No|\bar{E}\right]\Pr\left[\bar{E}\right]} = \frac{\Pr\left[No|E\right]q}{\Pr\left[No|E\right]q + 1 - q} \leq q$$

Therefore, for

$$\begin{aligned}
\frac{1}{6}\sigma_w\sqrt{\frac{2r_N^*}{rR_N^*}} - -c_N\left(r_N^*\right) &\geq 0 \qquad (7.6) \\
\frac{q}{6}\sigma_w\sqrt{\frac{2r_N^*}{rR_N^*}} - -c_N\left(r_N^*\right) &< 0 \qquad (7.7)
\end{aligned}$$

the pair $T_A = (\textit{conceal}, \overline{S_A})$ and $T_N = S_N$ and the pair $T_A = (\textit{falsify}, \overline{S_A})$ and $T_N = S_N$ are not the Nash equilibria, while the pair $T_A = (\textit{announce}, \overline{S_A})$ and $T_N = S_N$ is the equilibrium. Notice that this outcome does not depend on the trader A's cost function $c_A(r_A)$, but it does depend on his risk aversion coefficient, a.

One can check that strategies $((\textit{conceal}, S_A); \overline{S_N})$and $((\textit{falsify}, S_A); \overline{S_N})$ are also the Nash equilibria, but condition 2 of the proposition makes sure that they are not Pareto optimal.□

Proof. Proposition 3.1: The proof is standard. We assume that $p(x) = \mu + \lambda_A x = \mathrm{E}\{V|x\}$. Trader A chooses r_A to maximize his expected utility

$$\max_{r_A} \mathrm{E}\{-\exp\{-a(-p(x)x + V(x+I) - c(r_A)\}\}$$

where x is chosen to maximize the conditional expectation

$$\max_x \mathrm{E}\{-\exp\{-a(-p(x)x + V(x+I))\}|S_A\}$$
$$\max_x \left\{-x(\mu + \lambda_A x) + \mathrm{E}\{V|S_A\}(x+I) - \frac{1}{2}a\,\mathrm{Var}\{V|S_A\}(x+I)^2\right\}$$

The first order condition (FOC) gives us

$$x = \frac{\mathrm{E}\{V|S_A\} - \mu}{2\lambda_A + a\,\mathrm{Var}\{V|S_A\}} - \frac{a\,\mathrm{Var}\{V|S_A\}\,I}{2\lambda_A + a\,\mathrm{Var}\{V|S_A\}}$$

Using (6.1) and (6.3) we get

$$\begin{aligned} \mathrm{E}\{V|S_A\} &= \mu + \frac{r_A}{r + r_A}(S_A - \mu) = \mu + \frac{r_A}{R_A}(S_A - \mu); \\ \mathrm{Var}\{V|S_A\} &= \frac{1}{r + r_A} = \frac{1}{R_A}; \\ x &= \frac{r_A(S_A - \mu)}{2\lambda_A R_A + a} - \frac{aI}{2\lambda_A R_A + a}. \end{aligned}$$

The market maker sets the price equal to the conditional expectation

$$
\begin{aligned}
\mathrm{E}\{V|x\} &= \mu + \frac{\mathrm{Cov}(V;x)}{\mathrm{Var}(x)}\left(x - \mathrm{E}(x)\right) \\
&= \mu + \frac{\frac{r_A}{r(2\lambda_A R_A + a)}}{\frac{R_A r_A^2}{r r_A (2\lambda_A R_A + a)^2} + \left(\frac{a}{2\lambda_A R_A + a}\right)^2 \frac{1}{r_I}} x
\end{aligned}
$$

$$
\lambda = \frac{r_A\left(2\lambda_A R_A + a\right)}{r_A R_A + \frac{a^2}{r r_I}}
$$

Solving for λ_A we get

$$
\lambda = \frac{a r_A}{\frac{a^2 r}{r_I} - r_A R_A}.
$$

In order to have the condition $\lambda \geq 0$ to hold, r_A must satisfy

$$
0 \leq r_A \leq \frac{-r + \sqrt{r^2 + 4r\frac{a^2}{r_I}}}{2} < \frac{a^2}{r_I}
$$

Plugging the expression for x into the formula for expected utility and using auxiliary formulas (6.5) and (6.6) we get the following

$$
\begin{aligned}
\max_{r_A} U_A = \; & \mathrm{E}\left[\mathrm{E}_{\;S_A}\left[\mathrm{E}_{\;V}\left[-\exp\left\{-a\left(-\left(\mu + \lambda\left(\frac{r_A(S_A-\mu)}{2\lambda_A R_A + a} - \frac{aI}{2\lambda_A R_A + a}\right)\right)\right.\right.\right.\right.\right. \times \\
& \times \left(\frac{r_A(S_A-\mu)}{2\lambda_A R_A + a} - \frac{aI}{2\lambda_A R_A + a}\right) + \\
& \left.\left.\left.\left.\left. + V\left(\frac{r_A(S_A-\mu)}{2\lambda_A R_A + a} + \frac{2\lambda I}{2\lambda_A R_A + a}\right) - c_A(r_A)\right)\right\}\right| S_A\right]\right| I\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathrm{E}\left[\mathrm{E}_{\;S_A}\left[-\exp\left\{-a\left(\frac{\left(r_A(S_A-\mu) + 2\lambda R_A I\right)^2}{2R_A\left(2\lambda_A R_A + a\right)} + \left(\mu - \lambda I\right) I - c_A(r_A)\right)\right\}\right| I\right]\right] \\
&= -\sqrt{\frac{r\left(2\lambda_A R_A + a\right)}{R_A\left(2\lambda_A r + a\right)}}\, \mathrm{E}\left[\exp\left\{-a\left(\mu I - \frac{a\lambda_A}{2\lambda_A r + a} I^2 - c_A(r_A)\right)\right\}\right]
\end{aligned}
$$

We have to compare this with trader A's utility when he stays uninformed and trades against the upward sloping pricing function. In that case

$$\begin{aligned}
\mathrm{E}\left\{V|\overline{S_A}\right\} &= \mu; \\
\mathrm{Var}\left\{V|\overline{S_A}\right\} &= \frac{1}{r}; \\
x &= -\frac{aI}{2\lambda_A r + a} \\
U_A &= \mathrm{E}\left[\exp\left\{-a\left(\mu I - \frac{a\lambda_A}{2\lambda_A r + a} I^2\right)\right\}\right].
\end{aligned}$$

Comparing the last expression with his utility when he becomes informed, we can conclude that the necessary condition for r_A^* to be the optimum is that there exists such r_A that

$$0 \le c_A(r_A) \le \frac{1}{2a}\ln\left[1 + \frac{ar_A}{r\left(2\lambda_A R_A + a\right)}\right]$$

or

$$0 \le c_A(r_A) \le \frac{1}{2a}\ln\left[1 + \frac{r_A\left(a^2 r - r_A R_A r_I\right)}{r\left(a^2 r + r_A R_A r_I\right)}\right] \le \frac{1}{2a}\ln\left[1 + \frac{r_A}{r}\right]$$

for $a^2 r - r_A R_A r_I > 0$.

Notice also that if $\lambda_A = 0$ is given, then the best response is

$$\begin{aligned}
\max_{r_A} U_A &= -\sqrt{\frac{r}{R_A}}\,\mathrm{E}\left[\exp\left\{-a\left(\mu I - c_A(r_A)\right)\right\}\right] \\
&= -\sqrt{\frac{r}{R_A}}\exp\left\{\frac{(a\mu)^2}{2r_I} - ac_A(r_A)\right\} \ge -\exp\left\{\frac{a^2\mu^2}{2r_I}\right\}.
\end{aligned}$$

if

$$c_A(r_A) < \frac{1}{2a}\ln\left(1 + \frac{r_A}{r}\right).$$

It means that if for every r_A

$$c_A(r_A) > \frac{1}{2a}\ln\left(1 + \frac{r_A}{r}\right),$$

then trader A should stay uninformed.

Finally, to get expression (3.1) we have to take the expectation

$$\mathrm{E}\left[\exp\left\{-a\mu I - \frac{a\lambda_A}{2\lambda_A r + a} I^2\right\}\right] = \frac{1}{\sqrt{1 - 2\left(\frac{a^2\lambda_A}{2\lambda_A r + a}\right)\frac{1}{r_I}}} \times \exp\left\{\frac{a^2\mu^2}{2r_I\left(1 - 2\left(\frac{a^2\lambda_A}{2\lambda_A r + a}\right)\frac{1}{r_I}\right)}\right\}$$

to get

$$U_A = -\sqrt{\frac{r\left(r_A R_A r_I + a^2 r\right)}{R_A\left(a^2\left(r - 2r_A\right) + r_A(r - r_A)r_I\right)}} \times \exp\left\{\frac{a^2\mu^2\left(a^2 r + r_A(r - r_A)r_I\right)}{2r_I\left(a^2\left(r - 2r_A\right) + r_A(r - r_A)r_I\right)} + ac_A(r_A)\right\}$$

□

Proof. Proposition 3.4: Since there are only two traders and both of them submit limit orders, their information sets are equivalent, that is, knowing $\{I;\ p\}$ or $\{S_N;\ p\}$ is equivalent to knowing $\{I;\ S_N\}$. Therefore, first we find traders' orders as functions of S_N and I and then rewrite them as the limit orders.

The market maker sets the price $p(y)$

$$p(y) = \mathrm{E}\left[V|y\right]$$

where $y = x + z$ is the sum of traders' orders. Trader A submits order x and trader N submits order z.

Trader N maximizes his expected profit by choosing his order, z

$$\max_z \mathrm{E}\left[\left(V - p(y)\right) z | S_N; I\right].$$

From FOC we get

$$z = \frac{r_N\left(S_N - \mu\right)}{2\lambda_N} - \frac{x}{2}. \tag{7.8}$$

Trader A maximizes his expected utility

$$\max_x \mathrm{E}\left\{-\exp\left\{-a\left(-p(y)x + V(x+I)\right)\right\}|S_N; I\right\}$$

which is equivalent to maximizing the certainty equivalent

$$\max_x \left(x+I\right)\mathrm{E}\left[V|S_N\right] - \frac{1}{2}a\left(x+I\right)^2 \mathrm{Var}\left[V|S_N\right] - (\mu + \lambda_N z)x - \lambda_N x^2$$

where

$$\begin{aligned}\mathrm{E}\left[V|S_N\right] &= \mu + \frac{r_N\left(S_N - \mu\right)}{R_N};\\ \mathrm{Var}\left[V|S_N\right] &= \frac{1}{R_N}.\end{aligned}$$

From FOC we get

$$x = \frac{r_N\left(S_N - \mu\right)}{2\lambda_N R_N + a} - \frac{a}{2\lambda_N R_N + a}I - \frac{\lambda_N}{2\lambda_N R_N + a}z. \tag{7.9}$$

Solving equations (7.8) and (7.9) together we get

$$\begin{aligned}z &= \frac{r_N\left(\lambda_N R_N + a\right)\left(S_N - \mu\right)}{\lambda_N R_N\left(3\lambda_N R_N + 2a\right)} + \frac{aI}{3\lambda_N R_N + 2a} && (7.10)\\ x &= \frac{r_N\left(S_N - \mu\right)}{\left(3\lambda_N R_N + 2a\right)} - \frac{2aI}{3\lambda_N R_N + 2a} && (7.11)\\ y &= x + z = \frac{r_N\left(2\lambda_N R_N + a\right)\left(S_N - \mu\right)}{\lambda_N R_N\left(3\lambda_N R_N + 2a\right)} - \frac{aI}{3\lambda_N R_N + 2a}\end{aligned}$$

Using $p(y) = \mu + \lambda_N y$ we can rewrite expressions for x and z as

$$x = \frac{R_N}{a + 2\lambda_N R_N}(p - \mu) - \frac{a}{(a + 2\lambda_N R_N)} I$$

and

$$z = \frac{r_N}{\lambda_N R_N}(S_N - \mu) - \frac{p - \mu}{\lambda_N}.$$

To find expression for λ_N we have to calculate $\mathrm{Cov}(y; v)$ and $\mathrm{Var}(y)$

$$\begin{aligned}
\mathrm{Cov}(y; v) &= \frac{r_N (2\lambda_N R_N + a)}{\lambda_N r R_N (3\lambda_N R_N + 2a)} \\
\mathrm{Var}(y) &= \frac{R_N}{r r_N}\left(\frac{r_N (2\lambda_N R_N + a)}{\lambda_N R_N (3\lambda_N R_N + 2a)}\right)^2 + \frac{1}{r_I}\left(\frac{a}{3\lambda_N R_N + 2a}\right)^2 \\
&= \frac{r_N}{r R_N}\left(\frac{(2\lambda_N R_N + a)}{\lambda_N (3\lambda_N R_N + 2a)}\right)^2 + \frac{1}{r_I}\left(\frac{a}{3\lambda_N R_N + 2a}\right)^2
\end{aligned}$$

Using Expression (6.2) we can write that

$$p(y) = \mu + \lambda_N y = \mathrm{E}[V|y] = \mu + \frac{\mathrm{Cov}(y; v)}{\mathrm{Var}(y)} y$$

and

$$\lambda_N = \frac{\frac{r_N (2\lambda_N R_N + a)}{\lambda_N r R_N (3\lambda_N R_N + 2a)}}{\frac{r_N}{r R_N}\left(\frac{(2\lambda_N R_N + a)}{\lambda_N (3\lambda_N R_N + 2a)}\right)^2 + \frac{1}{r_I}\left(\frac{a}{3\lambda_N R_N + 2a}\right)^2}$$

Rearranging terms and simplifying we get the following quadratic equation

$$\left(2 r_N R_N^2 r_I - a^2 r R_N\right)\lambda_N^2 + (3 a r_N R_N r_I)\lambda_N + a^2 r_N r_I = 0$$

which has the positive solution

$$\lambda_N = \frac{3 a r_N r_I + a\sqrt{(r_N r_I)^2 + 4a^2 \frac{r r_N r_I}{R_N}}}{2\left(a^2 r R_N - 2 r_N r_I R_N^2\right)}$$

provided that

$$a^2 r R_N - 2 r_N r_I R_N^2 > 0$$

which means that

$$0 \le r_N < \frac{-r + \sqrt{r^2 + 2\frac{a^2 r}{r_I}}}{2} < \frac{a^2}{2 r_I}$$

$\square$.

Proof. **Corollary 3.6**: The proof is straightforward. If trader N does not trade, his expected profit is zero. If he decides to stay uninformed and benefit from trader A's risk-sharing trade, while trader A assumes that he receives a signal S_N, it is equivalent to

$$x = -\frac{2aI}{3\lambda_N R_N + 2a}$$

and

$$z = \frac{aI}{3\lambda_N R_N + 2a}.$$

These formulae are obtained from (7.10) and (7.11) by setting $S_N = \mu$. The price becomes equal to

$$p = \mu - \lambda \frac{aI}{3\lambda_N R_N + 2a}$$

and trader N makes a profit

$$\Pi_N = (\mu - p)\, z = \lambda \left(\frac{aI}{3\lambda_N R_N + 2a} \right)^2$$

His expected profit is

$$\mathrm{E}\,\Pi_N = \frac{\lambda}{r_I} \left(\frac{a}{3\lambda_N R_N + 2a} \right)^2.$$

On the other hand, if trader acquires signal S_N at cost $c_N(r_N)$ then

$$\begin{aligned} z &= \frac{r_N\left(S_N-\mu\right)\left(\lambda_N R_N+a\right)}{\lambda_N R_N\left(3\lambda_N R_N+2a\right)}+\frac{aI}{3\lambda_N R_N+2a} \\ p &= \mu+\lambda\left(\frac{r_N\left(2\lambda_N R_N+a\right)\left(S_N-\mu\right)}{\lambda_N R_N\left(3\lambda_N R_N+2a\right)}-\frac{aI}{3\lambda_N R_N+2a}\right) \end{aligned}$$

and his profit becomes

$$\Pi_N=\frac{1}{\lambda}\left(\frac{r_N\left(\lambda_N R_N+a\right)\left(S_N-\mu\right)}{R_N\left(3\lambda_N R_N+2a\right)}+\frac{aI}{3\lambda_N R_N+2a}\right)^2-c_N(r_N)$$

and his expected profit is

$$\mathrm{E}\,\Pi_N=\frac{1}{\lambda}\left[\frac{r_N\left(a+\lambda R_N\right)^2}{rR_N\left(2a+3\lambda R_N\right)^2}+\frac{(a\lambda)^2}{r_I\left(2a+3\lambda R_N\right)^2}\right]-c_N(r_N)$$

which establishes the result.□

Proof. Proposition ??: We denote by $\mathrm{E}\left\{U_i\left(T_A;T_N\right)\right\}$, $i=\{A;\ N\}$ trader's i expected utility when strategies T_A and T_N are played. Then the pair of strategies $T_A=($*announce*, $\overline{S_A})$ and $T_N=S_N$ is the Nash equilibrium if the following inequalities hold:

$$\begin{aligned} \mathrm{E}\left\{U_A((announce,\overline{S_A});S_N)\right\} &\geq \mathrm{E}\left\{U_A((conceal,\overline{S_A});S_N)\right\} \\ \mathrm{E}\left\{U_A((announce,\overline{S_A});S_N)\right\} &\geq \mathrm{E}\left\{U_A((falsify,\overline{S_A});S_N)\right\} \\ \mathrm{E}\left\{U_A((announce,\overline{S_A});S_N)\right\} &\geq \mathrm{E}\left\{U_A((conceal,S_A);S_N)\right\} \\ \mathrm{E}\left\{U_A((announce,\overline{S_A});S_N)\right\} &\geq \mathrm{E}\left\{U_A((falsify,S_A);S_N)\right\} \\ \mathrm{E}\left\{U_A((announce,\overline{S_A});S_N)\right\} &\geq \mathrm{E}\left\{U_A((announce,S_A);S_N)\right\} \\ \mathrm{E}\left\{U_N((announce,\overline{S_A});S_N)\right\} &\geq \mathrm{E}\left\{U_N((announce,\overline{S_A});\overline{S_N})\right\} \end{aligned}$$

The first two expressions are equalities. Expressions 3-5 are strict inequalities, because of the information cost borne by trader A and the nature of the signal $S_A=S_N+\varepsilon_{A/N}$.

The sixth inequality holds whenever

$$\begin{aligned}
\mathrm{E}\left\{\mathrm{E}\left\{U_N(\cdot)|r_N\right\}\right\} - c_N(r_N) &\geq \mathrm{E}\left\{\mathrm{E}\left\{U_N(\cdot)|x\right\}\right\} \\
\mathrm{E}\left\{U_N(\cdot)|event; r_N\right\} - c_N(r_N) &\geq \left\{U_N(\cdot)|event; x\right\}
\end{aligned}$$

The pair of strategies $T_A = (conceal, \overline{S_A})$ and $T_N = S_N$ is the Nash equilibrium if the following inequalities hold:

$$\begin{aligned}
\mathrm{E}\left\{U_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{U_A((announce, \overline{S_A}); S_N)\right\} \\
\mathrm{E}\left\{U_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{U_A((falsify, \overline{S_A}); S_N)\right\} \\
\mathrm{E}\left\{U_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{U_A((announce, S_A); S_N)\right\} \\
\mathrm{E}\left\{U_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{U_A((falsify, S_A); S_N)\right\} \\
\mathrm{E}\left\{U_A((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{U_A((conceal, S_A); S_N)\right\} \\
\mathrm{E}\left\{U_N((conceal, \overline{S_A}); S_N)\right\} &\geq \mathrm{E}\left\{U_N((conceal, \overline{S_A}); \overline{S_N})\right\}
\end{aligned}$$

The first two expressions are equalities. Expressions 3-5 are strict inequalities, because of the information cost $c_A(r_A)$. The last one holds whenever

$$\begin{aligned}
\mathrm{E}\left\{\mathrm{E}\left\{U_N(\cdot)|r_N\right\}\right\} - c_N(r_N) &\geq \mathrm{E}\left\{\mathrm{E}\left\{U_N(\cdot)|x\right\}\right\} \\
q\,\mathrm{E}\left\{U_N(\cdot)|event; r_N\right\} - c_N(r_N) &\geq q\left\{U_N(\cdot)|event; x\right\}
\end{aligned}$$

We also have to check the pair $T_A =$ (*falsify*, $\overline{S_A}$) and $T_N = S_N$. Since trader A has no incentive to lie when the event does not actually happen, he can possibly falsify only when the innovation has taken place. The probability of $E = \{innovation\}$ when trader A says that there is no innovation is

$$\Pr[E|No] = \frac{\Pr[No|E]\Pr[E]}{\Pr[No|E]\Pr[E] + \Pr\left[No|\bar{E}\right]\Pr\left[\bar{E}\right]} = \frac{\Pr[No|E]\,q}{\Pr[No|E]\,q + 1 - q} \leq q$$

Therefore, for

$$\begin{aligned}
\mathrm{E}\{U_N(\cdot)|E;r_N\} - c_N(r_N) &\geq \mathrm{E}\{U_N(\cdot)|E;x\} &(7.12)\\
\mathrm{E}\{U_N(\cdot)|E;x\} + c_N(r_N) &< \mathrm{E}\{U_N(\cdot)|E;r_N\} < \frac{\mathrm{E}\{U_N(\cdot)|E;x\} + c_N(r_N)}{q} &(7.13)
\end{aligned}$$

the pair $T_A = (\textit{conceal}, \overline{S_A})$ and $T_N = S_N$ and the pair $T_A = (\textit{falsify}, \overline{S_A})$ and $T_N = S_N$ are not the Nash equilibria, while the pair $T_A = (\textit{announce}, \overline{S_A})$ and $T_N = S_N$ is the equilibrium. Notice that this outcome does not depend on the trader A's cost function $c_A(r_A)$, but it does depend on his risk aversion coefficient, a.

We also have to check the strategy profiles $((\textit{conceal}, \overline{S_A}); \overline{S_N})$, $((\textit{conceal}, S_A); \overline{S_N})$, $((\textit{falsify}, \overline{S_A}); \overline{S_N})$ and $((\textit{falsify}, S_A); \overline{S_N})$ as remaining candidates to be the pure Nash linear equilibrium. Because of inequalities (7.12) and (7.13) the market maker knows that when no announcement is made trader N remains uninformed. If the limit order contains a signal component, it can come only from trader A. The problem is that with

$$B \leq c_A(r_A) < \frac{1}{2a} \ln\left(1 + \frac{r_A}{r}\right)$$

where

$$B = \frac{1}{2a} \ln\left[1 + \frac{r_A\left(a^2 r - r_A R_A r_I\right)}{r\left(a^2 r + r_A R_A r_I\right)}\right]$$

for trader A it is too costly to acquire the signal S_A if $\lambda_A \neq 0$, but is too cheap to restrain trader A from acquiring the signal if $\lambda_A = 0$. Therefore these two pairs of strategies cannot be part of a linear Nash equilibrium.$\square$

Notes

[1]We can offer several explanations for this liquidity shock. For example, trader A is an agent of a mutual fund, which holds other risky securities that are not part of our model. When the innovation occurs it affects the variance-covariance matrix of the securities in the fund's portfolio and the portfolio has to be rebalanced for this reason.

[2]Sunshine trading is when liquidity traders preannounce their liquidity driven trades and the market maker believes them (see Admati and Pfleiderer (1991)).

References

Admati, A., and P. Pfleiderer, 1986, "A Monopolistic Market for Information," *Journal of Economic Theory*, 39, 400–438.

——— , 1987, "Viable Allocations of Information in Financial Markets," *Journal of Economic Theory*, 43, 76–115.

——— , 1988, "Selling and Trading on Information in Financial Markets," *American Economic Review*, 78(2), 96–103.

——— , 1991, "Sunshine Trading and Financial Market Equilibrium," *Review of Financial Studies*, 4(3), 443–481.

——— , 2000, "Forcing Firms to Talk: Financial Disclosure Regulations and Externalities," *Review of Financial Studies*, 13(3), 479–519.

Bhattacharya, U., and M. Spiegel, 1991, "Insiders, Outsiders and Market Breakdowns," *Review of Financial Studies*, 2(4), 255–282.

Biais, B., and J. Rochet, 1996, "Risk Sharing, Adverse Selection and Market Structure," in *Financial Mathematics (Lecture Notes in Mathematics, 1656)*, ed. by W. Runggaldier. Springer, chap. 1, pp. 1–52.

Boot, A. W., and A. V. Thakor, 2001, "The Many Faces of Information Disclosure," *Review of Financial Studies*, 14.

Brunnermeier, M. K., 1998, "Buy on Rumors - Sell on News: A Manipulative Trading Strategy," FMG Discussion Paper 309.

Crawford, V., and J. Sobel, 1982, "Strategic Information Transmission," *Econometrica*, 50(6), 1431–1451.

Foster, F., and S. Viswanathan, 1996, "Strategic Trading When Agents Forecast the Forecasts of Others," *Journal of Finance*, 51(4), 1437–1477.

Gal-Or, E., 1985, "Information Sharing in Oligopoly," *Econometrica*, 53(2), 329–343.

——— , 1986, "Information Transmission - Cournot and Bertrand Equilibria," *Review of Economic Studies*, 53, 85–92.

Glosten, L., 1989, "Insider Trading, Liquidity and the Role of the Monopolist Specialist," *Journal of Business*, 62(2), 211–235.

Greene, W., 1993, *Econometric Analysis*. Prentice-Hall.

Grossman, S., and J. Stiglitz, 1980, "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70, 393–408.

Kyle, A., 1985, "Continuous Actions and Insider Trading," *Econometrica*, 53(6), 1315–1335.

Medrano, L., and X. Vives, 1997, "Strategic Behavior and Price Discovery," *CEPR Discussion Paper*, (1768).

Persico, N., 1995, "Information Acquisition in Affiliated Problems," working paper, University of Pennsylvania.

Verrecchia, R. E., 1982, "Information Acquisition in a Noisy Rational Expectations Economy," *Econometrica*, 50(6), 1415–1430.

Vives, X., 1985, "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal of Economic Theory*, 34, 71–94.

8. Figures

Figure 1 shows the game between trader A and trader N. Innovation takes place with probability q. After receiving a coarse signal, trader A makes an announcement decision. Then traders can buy signals about the value of innovation. The dotted lines connect the nodes which belong to the same information set for a given trader.

Figure 2 displays Kyle's market depth (price sensitivity) λ_A (dotted line) and risk-averse trader A's utility (solid line) as functions of signal precision, r_A, when trader A does not make an announcement about the innovation, acquires information himself and trades alone. $c_A = 0$, $r = 3$, $r_I = 2$, $\mu = 1$, $a = 1.5$.Even with zero signal acquisition cost, trader A's utility is strictly decreasing as the function of r_A. The critical value for r_A is $r_A^c = .8717$. This is the value at which λ_A would become infinite.

Figure 3 shows regions where different decisions about information acquisition are made by risk-averse trader A depending on the signal cost and signal precision r_A when he does not make an announcement about the innovation. In the top left corner, trader A remains uninformed and the market maker anticipates this. The lower region correspond to the equilibrium outcome where trader A becomes informed. No linear equilibrium is possible in the region between these two. $r = 3$, $r_I = 2$, $\mu = 1$, $a = 1.5$

Figure 4 displays Kyle's market depth (price sensitivity) λ (solid line) and trader N's profit Π_N (dotted line) as functions of signal precision, r_N. $c_N = 0$, $r = 3$, $r_I = 2$, $\mu = 1$, $a = 1.5$. We can see that λ_N is increasing with r_N while Π_N initially increases, then reaches its maximum at $r_N = .275$ and starts decreasing. Its decrease for high values of r_N can be explained by the high value of λ_N and the relatively low level of "noise trading" determined by volatility of the endowment shock I. The critical value, $r_N^c = .4843$, which is smaller than the value r_A^c.

Figure 5 shows utilities of trader A for different possible outcomes. The downward sloping solid curve is his utility if he announces. It is a function of information precision, r_N, chosen by trader N. The downward sloping dotted line is trader's A utility when he conceals information and becomes informed himself. The dashed horizontal line is his utility from trading uninformed, when sunshine trading is possible ($\lambda = 0$) and the upward sloping line is his utility when he deviates and becomes informed while the market maker keeps $\lambda = 0$. Information acquisition costs are zero. Stars mark the point corresponding to precision, r_N, when trader N's profit reaches the maximum. $c = 0$, $r = 3$, . $r_I = 2$, $\mu = 1$, $a = 1.5$

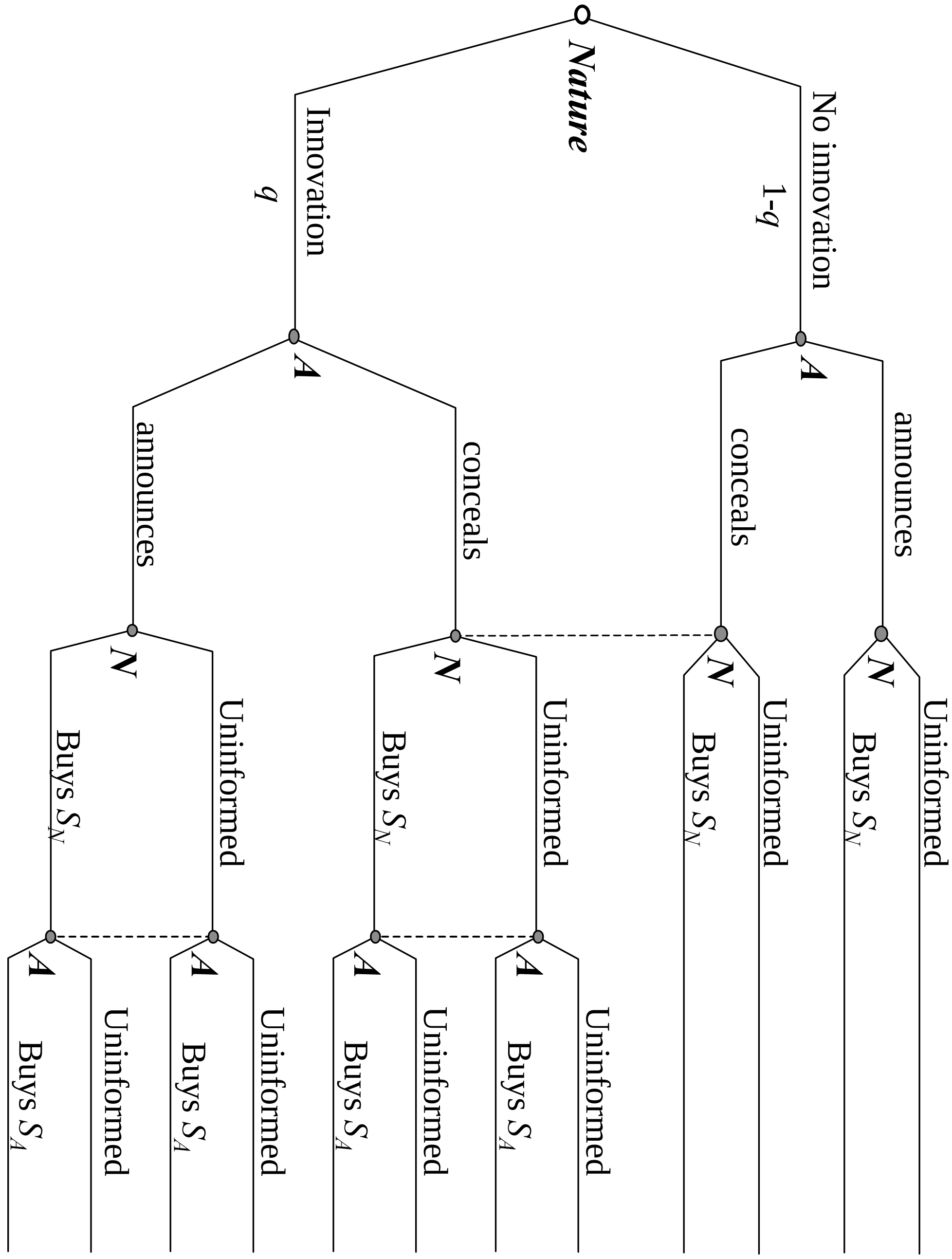
Nature
Innovation
q
No innovation
1-q
A
announces
conceals
A
conceals
announces
N
Buys S_N
Uninformed
N
Buys S_N
Uninformed
N
Buys S_N
Uninformed
N
Buys S_N
Uninformed
A
Buys S_A
Uninformed
A
Buys S_A
Uninformed
A
Buys S_A
Uninformed
A
Buys S_A
Uninformed

Figure 1:

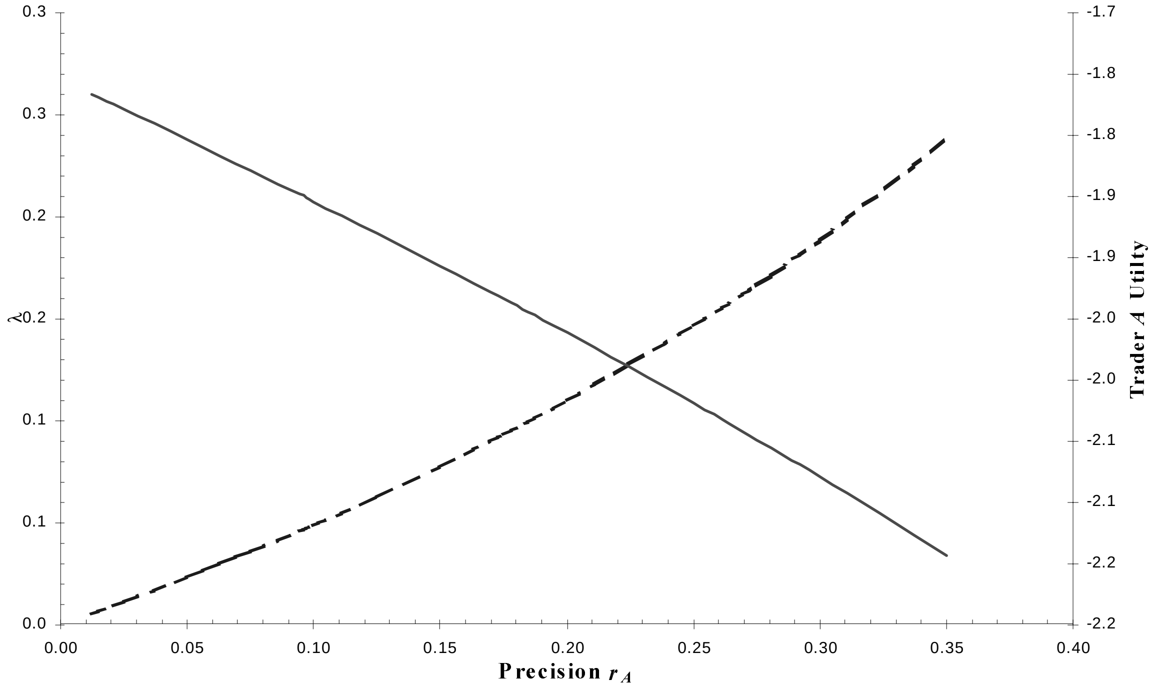
0.3
0.3
0.2
λ 0.2
0.1
0.1
0.0
-1.7
-1.8
-1.8
-1.9
-1.9
-2.0
-2.0
-2.1
-2.1
-2.2
-2.2
Trader A Utilty
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
Precision r_A

Figure 2:

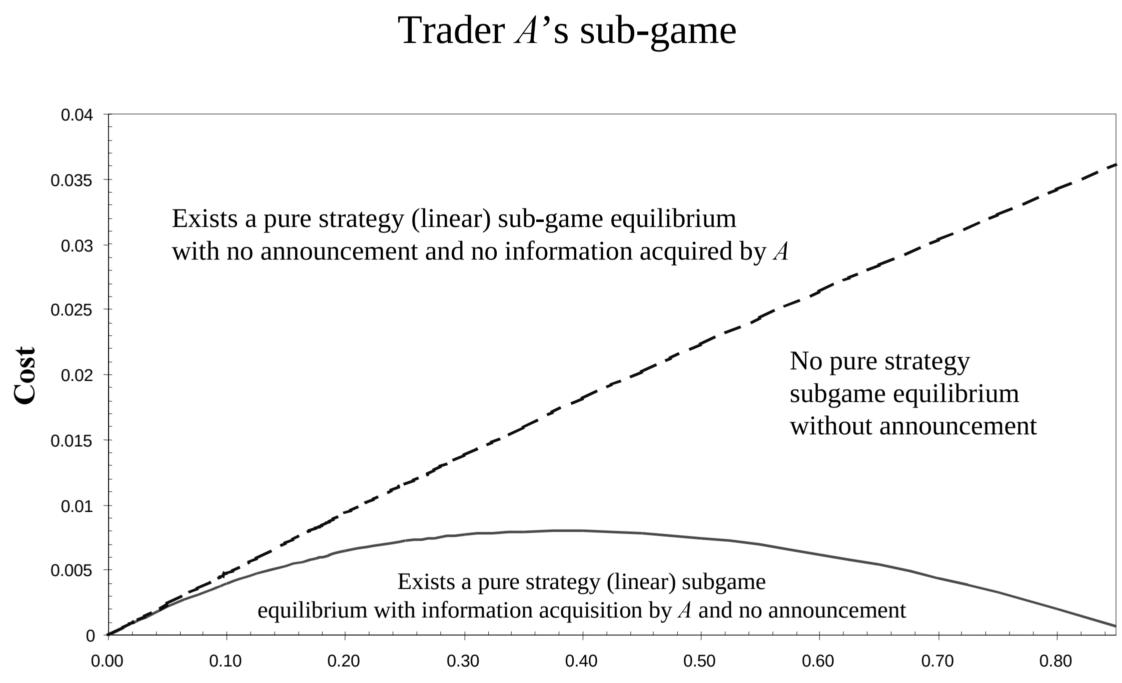
Trader A's sub-game
Cost
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
Exists a pure strategy (linear) sub-game equilibrium with no announcement and no information acquired by A
No pure strategy subgame equilibrium without announcement
Exists a pure strategy (linear) subgame equilibrium with information acquisition by A and no announcement

Figure 3:

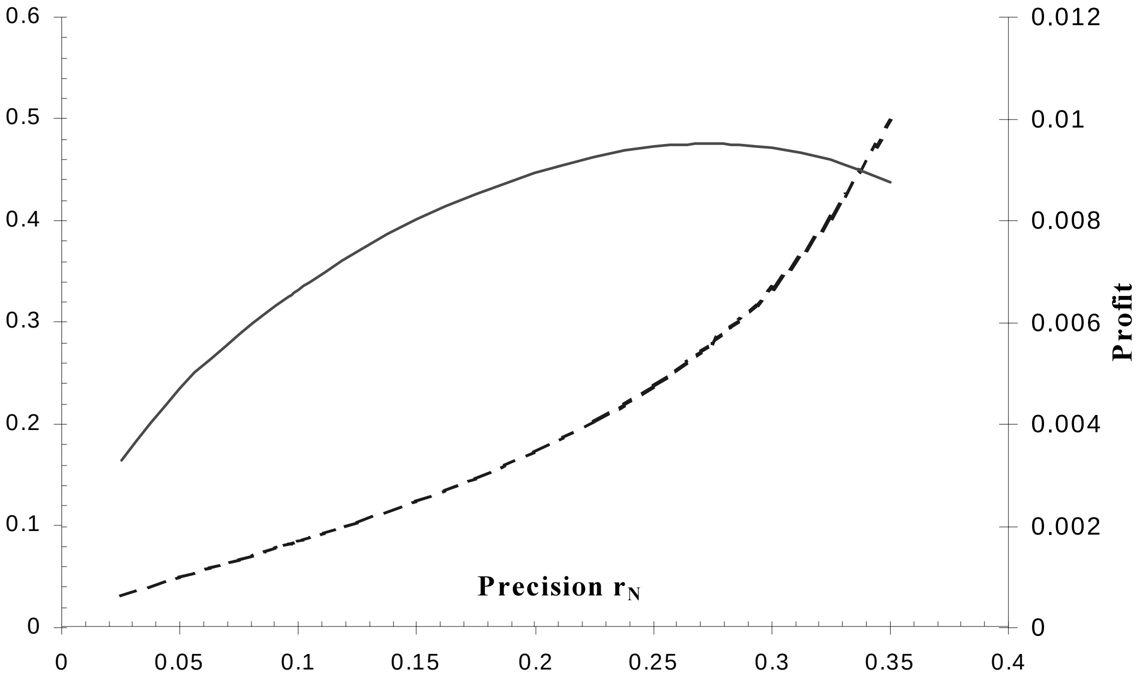
0.6
0.5
0.4
0.3
0.2
0.1
0
0.012
0.01
0.008
0.006
0.004
0.002
0
Profit
Precision r_N
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

Figure 4:

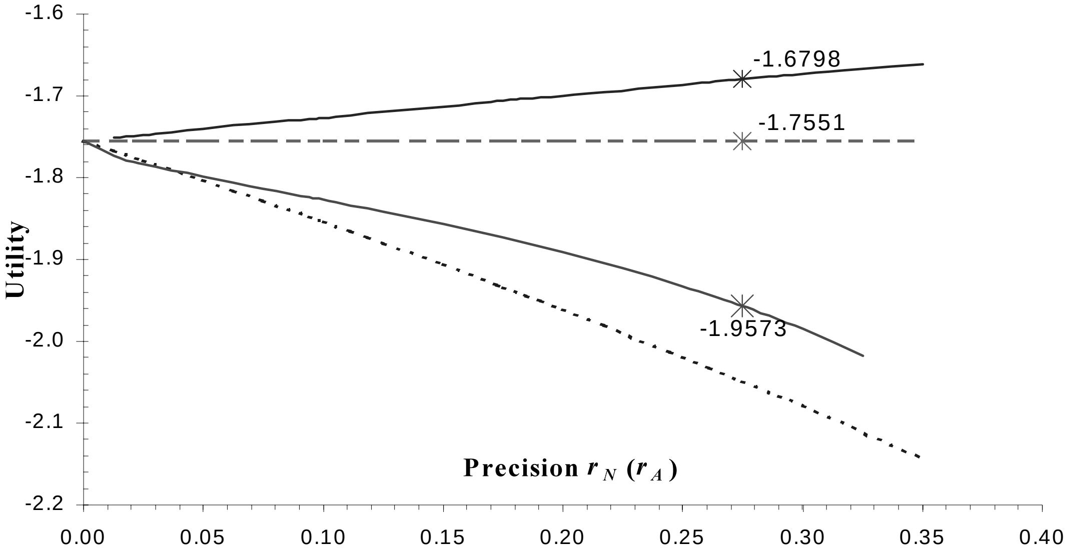
-1.6
-1.7
-1.8
-1.9
-2.0
-2.1
-2.2
Utility
-1.6798
-1.7551
-1.9573
Precision r_N (r_A)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

Figure 5: